小テスト第 9 回解答　2015/07/06

小テスト第 9 回

1.

$$\frac{\partial g}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} = \frac{\partial f}{\partial u} + \frac{\partial f}{\partial v}$$
$$\frac{\partial g}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} = \frac{\partial f}{\partial u} - \frac{\partial f}{\partial v}$$

ただし、$g(x,y)$ は x,y 変数の関数だから、上の式の左辺は (もちろん、右辺も) x,y の関数である ことに注意せよ。丁寧に書けば、($u = \varphi(x,y) = x+y$, $v = \psi(x,y) = x-y$ として)

$$\begin{aligned}\frac{\partial g}{\partial x}(x,y) &= \frac{\partial f}{\partial u}(\varphi(x,y),\psi(x,y))\frac{\partial \varphi}{\partial x}(x,y) + \frac{\partial f}{\partial v}(\varphi(x,y),\psi(x,y))\frac{\partial \psi}{\partial x}(x,y)\\ &= \frac{\partial f}{\partial u}(x+y,x-y) + \frac{\partial f}{\partial v}(x+y,x-y)\\ \frac{\partial g}{\partial y}(x,y) &= \frac{\partial f}{\partial u}(\varphi(x,y),\psi(x,y))\frac{\partial \varphi}{\partial y}(x,y) + \frac{\partial f}{\partial v}(\varphi(x,y),\psi(x,y))\frac{\partial \psi}{\partial y}(x,y)\\ &= \frac{\partial f}{\partial u}(x+y,x-y) - \frac{\partial f}{\partial v}(x+y,x-y)\end{aligned}$$

となる。

2. (1) $\dfrac{\partial f}{\partial u} = e^{u+v} + e^{u-v}$, $\dfrac{\partial f}{\partial v} = e^{u+v} - e^{u-v}$

(2) 問 1. でも書いたように、$\dfrac{\partial g}{\partial x}$, $\dfrac{\partial g}{\partial y}$ が x,y の関数であることに注意する。(2) より、$\dfrac{\partial f}{\partial x}(x+y,x-y) = e^{2x} + e^{2y}$, $\dfrac{\partial f}{\partial y}(x+y,x-y) = e^{2x} - e^{2y}$ となるから、$\dfrac{\partial g}{\partial x} = 2e^{2x}$, $\dfrac{\partial g}{\partial y} = 2e^{2y}$.