

ひと目の積分 問題編

- ▷ 積分計算の基礎力を付ける+aの問題集です。典型的な問題を取り上げてみました。積分の問題を解く呼吸を思い出しましょう。
- ▷ まずは、微分の問題をひと目で解いてください。微分ができなければ積分はできません。
- ▷ 積分の問題をひと目見て、解き方がわかるものは計算する必要がないと思います。
- ▷ ひと目見てわからない問題は、ノートに書いて計算してみましょう。
- ▷ 考えてわからない問題は解答を見て、読んだら、解答を伏せてもう一度。
- ▷ すべての問題がひと目で解き方がわかるようになるまで何度も取り組んでみましょう。

微分編

- (1) $\tan x$ (2) $\text{Arcsin } 2x$ (3) e^{-x^2} (4) 3^x
- (5) $\cos 3x$ (6) $\text{Arctan } \sqrt{3}x$ (7) $x^{\frac{1}{3}}$ (8) $\log(3x+1)$

基本積分

- (1) $\int \frac{1}{\sqrt{x+1}} dx$ (2) $\int \frac{1}{x^2+1} dx$ (3) $\int \frac{1}{(x+1)^2} dx$ (4) $\int \frac{1}{\sqrt{1-x^2}} dx$
- (5) $\int 3^x dx$ (6) $\int x^{\frac{1}{3}} dx$ (7) $\int \frac{1}{\sqrt{3x-\sqrt{3x+2}}} dx$ (8) $\int \cos^2 x dx$

部分積分

- (1) $\int x \sin x dx$ (2) $\int x^2 e^x dx$ (3) $\int x^2 2^x dx$ (4) $\int \sqrt{x} \log x dx$
- (5) $\int \log x dx$ (6) $\int (\log x)^2 dx$ (7) $\int \frac{\log x}{x^2} dx$ (8) $\int e^x \cos x dx$

置換積分

- (1) $\int e^{\sin x} \cos x dx$ (2) $\int x \sin x^2 dx$ (3) $\int e^{\sqrt{x}} dx$ (4) $\int \frac{e^x}{\cos^2 e^x} dx$
- (5) $\int \tan x dx$ (6) $\int (\log x)^2 dx$ (7) $\int \frac{1}{\tan x} dx$ (8) $\int \frac{\sin x}{\sqrt{1-\cos x}} dx$

log 型 † $\int \frac{f'(x)}{f(x)} dx = \log|f(x)|$

- (1) $\int \frac{e^{-x}}{e^{-x}+4} dx$ (2) $\int \frac{x}{3x^2+2} dx$ (3) $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ (4) $\int \tan x dx$
- (5) $\int \frac{x+3}{x^2+6x+5} dx$ (6) $\int \frac{1}{x \log x} dx$ (7) $\int \frac{1}{\tan x} dx$ (8) $\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+2} dx$

Arctan 型 $\int \frac{1}{1+x^2} dx = \text{Arctan } x$

- (1) $\int \frac{1}{4+x^2} dx$ (2) $\int \frac{1}{(x-1)^2+2} dx$ (3) $\int \frac{1}{x^2+4x+8} dx$ (4) $\int \frac{1}{2x^2-4x+7} dx$

Arcsin 型 $\int \frac{1}{\sqrt{1-x^2}} dx = \text{Arcsin } x$

- (1) $\int \frac{1}{\sqrt{4-x^2}} dx$ (2) $\int \frac{1}{\sqrt{-(x+1)^2+2}} dx$ (3) $\int \frac{1}{\sqrt{-x^2+4x-1}} dx$ (4) $\int \frac{1}{\sqrt{-2x^2+4x+1}} dx$

† この形の積分のことを便宜的に log 型と書きました。置換積分の特別な形に過ぎませんが、この形を知っておくと計算が楽に早くできます:

$$\int \frac{f'(x)}{f(x)} dx = \int \frac{1}{u} du = \log|f(x)| \quad \begin{array}{l} u = f(x) \\ du = f'(x) dx \end{array}$$

ひと目の積分 解答編

微分編

$$(1) \frac{1}{\cos^2 x}$$

$$(2) \frac{2}{\sqrt{1-4x^2}}$$

$$(3) -2xe^{-x^2}$$

$$(4) 3^x \log 3$$

$$(5) -3 \sin 3x$$

$$(6) \frac{\sqrt{3}}{1+3x^2}$$

$$(7) \frac{1}{3}x^{-\frac{2}{3}}$$

$$(8) \frac{3}{3x+1}$$

基本積分

$$(1) \int \frac{1}{\sqrt{x+1}} dx = 2\sqrt{x+1}$$

$$(2) \int \frac{1}{x^2+1} dx = \text{Arctan } x$$

$$(3) \int \frac{1}{(x+1)^2} dx = -\frac{1}{x+1}$$

$$(4) \int \frac{1}{\sqrt{1-x^2}} dx = \text{Arcsin } x$$

$$(5) \int 3^x dx = \frac{1}{\log 3} 3^x$$

$$(6) \int x^{\frac{1}{3}} dx = \frac{3}{4}x^{\frac{4}{3}}$$

$$(7) \int \frac{1}{\sqrt{3x}-\sqrt{3x+2}} dx = \int \frac{\sqrt{3x}+\sqrt{3x+2}}{(3x)-(3x+2)} dx = -\frac{1}{9}(3x)^{\frac{3}{2}} - \frac{1}{9}(3x+2)^{\frac{3}{2}}$$

$$(8) \int \cos^2 x dx = \int \frac{\cos 2x + 1}{2} dx = \frac{1}{4} \sin 2x + \frac{1}{2}x$$

部分積分

$$(1) \int x \sin x dx = \int x(-\cos x)' dx = x(-\cos x) - \int 1 \cdot (-\cos x) dx = -x \cos x + \sin x$$

$$(2) \int x^2 e^x dx = x^2 e^x - \int 2x \cdot e^x dx = x^2 e^x - (2x \cdot e^x - \int 2 \cdot e^x dx) = x^2 e^x - 2x e^x + 2e^x$$

$$(3) \int x^2 2^x dx = x^2 \cdot \frac{2^x}{\log 2} - \int 2x \cdot \frac{2^x}{\log 2} dx = x^2 \cdot \frac{2^x}{\log 2} - \left(2x \cdot \frac{2^x}{(\log 2)^2} - \int 2 \cdot \frac{2^x}{(\log 2)^2} dx \right) = x^2 \cdot \frac{2^x}{\log 2} - 2x \cdot \frac{2^x}{(\log 2)^2} + 2 \cdot \frac{2^x}{(\log 2)^3}$$

$$(4) \int \sqrt{x} \log x dx = \frac{2}{3}x^{\frac{3}{2}} \log x - \int \frac{2}{3}x^{\frac{3}{2}} \cdot \frac{1}{x} dx = \frac{2}{3}x^{\frac{3}{2}} \log x - \frac{4}{9}x^{\frac{3}{2}}$$

$$(5) \int \log x dx = \int (x)' \log x dx = x \log x - \int x \cdot \frac{1}{x} dx = x \log x - x$$

$$(6) \int (\log x)^2 dx = x(\log x)^2 - \int x \cdot 2 \log x \cdot \frac{1}{x} dx = x(\log x)^2 - 2 \int \log x dx = x(\log x)^2 - 2x \log x + 2x$$

$$(7) \int \frac{\log x}{x^2} dx = \int \left(-\frac{1}{x} \right)' \log x dx = -\frac{1}{x} \log x - \int \left(-\frac{1}{x} \right) \cdot \frac{1}{x} dx = -\frac{1}{x} \log x - \frac{1}{x}$$

$$(8) I = \int e^x \cos x dx = e^x \cos x - \int e^x \cdot (-\sin x) dx = e^x \cos x + e^x \sin x - I \quad \therefore I = \frac{e^x \cos x + e^x \sin x}{2}$$

置換積分

$$(1) \int e^{\sin x} \cos x dx = \int e^u du = e^{\sin x} \quad \begin{array}{l} u = \sin x \\ du = \cos x dx \end{array}$$

$$(2) \int x \sin x^2 dx = \int \frac{1}{2} \cos u du = -\frac{1}{2} \cos x^2 \quad \begin{array}{l} u = x^2 \\ du = 2x dx \end{array}$$

$$(3) \int e^{\sqrt{x}} dx = \int e^u \cdot 2u du = 2ue^u - 2e^u = 2\sqrt{x} e^{\sqrt{x}} - 2e^{\sqrt{x}} \quad \begin{array}{l} u = \sqrt{x} \\ du = \frac{1}{2}x^{-\frac{1}{2}} dx = \frac{1}{2u} dx \end{array}$$

$$(4) \int \frac{e^x}{\cos^2 e^x} dx = \int \frac{u}{\cos^2 u} \cdot \frac{1}{u} du = \tan u = \tan e^x \quad \begin{array}{l} u = e^x \\ du = e^x dx = u dx \end{array}$$

$$(5) \int \tan x dx = \int \frac{\sin x}{\cos x} dx = \int -\frac{1}{u} du = -\log|\cos x| \quad \begin{array}{l} u = \cos x \\ du = -\sin x dx \end{array}$$

$$(6) \int (\log x)^2 dx = \int u^2 \cdot e^u du = u^2 e^u - 2ue^u + 2e^u = x(\log x)^2 - 2x \log x + 2x \quad \begin{array}{l} u = \log x \\ du = \frac{1}{x} dx = e^{-u} dx \end{array}$$

$$(7) \int \frac{1}{\tan x} dx = \int \frac{\cos x}{\sin x} dx = \int \frac{1}{u} du = \log|\sin x| \quad \begin{array}{l} u = \sin x \\ du = \cos x dx \end{array}$$

$$(8) \int \frac{\sin x}{\sqrt{1-\cos x}} dx = \int \frac{1}{\sqrt{u}} du = 2\sqrt{u} = 2\sqrt{1-\cos x} \quad \begin{array}{l} u = 1 - \cos x \\ du = \sin x dx \end{array}$$

$$\log \text{型} \quad \int \frac{f'(x)}{f(x)} dx = \log|f(x)|$$

$$(1) \int \frac{e^{-x}}{e^{-x}+4} dx = -\int \frac{(e^{-x}+4)'}{e^{-x}+4} dx = -\log(e^{-x}+4)$$

$$(2) \int \frac{x}{3x^2+2} dx = \frac{1}{6} \int \frac{6x}{3x^2+2} dx = \frac{1}{6} \log(3x^2+2)$$

$$(3) \int \frac{\sin x - \cos x}{\sin x + \cos x} dx = -\log|\sin x + \cos x|$$

$$(4) \int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\log|\cos x|$$

$$(5) \int \frac{x+3}{x^2+6x+5} dx = \frac{1}{2} \int \frac{2x+6}{x^2+6x+5} dx = \frac{1}{2} \log|x^2+6x+5|$$

$$(6) \int \frac{1}{x \log x} dx = \int \frac{\frac{1}{x}}{\log x} dx = \log|\log x|$$

$$(7) \int \frac{1}{\tan x} dx = \int \frac{\cos x}{\sin x} dx = \log|\sin x|$$

$$(8) \int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+2} dx = \frac{1}{\sqrt{2}} \log(x^2+\sqrt{2}x+2)$$

$$\text{Arctan 型} \quad \int \frac{1}{1+x^2} dx = \text{Arctan } x$$

$$(1) \int \frac{1}{4+x^2} dx = \frac{1}{4} \int \frac{1}{1+(\frac{x}{2})^2} dx = \frac{1}{4} \text{Arctan } \frac{x}{2} \times 2 = \frac{1}{2} \text{Arctan } \frac{x}{2}$$

$$(2) \int \frac{1}{(x-1)^2+2} dx = \frac{1}{2} \int \frac{1}{(\frac{x-1}{\sqrt{2}})^2+1} dx = \frac{1}{2} \text{Arctan } \frac{x-1}{\sqrt{2}} \times \sqrt{2} = \frac{1}{\sqrt{2}} \text{Arctan } \frac{x-1}{\sqrt{2}}$$

$$(3) \int \frac{1}{x^2+4x+8} dx = \int \frac{1}{(x+2)^2+4} dx = \frac{1}{4} \int \frac{1}{(\frac{x+2}{2})^2+1} dx = \frac{1}{4} \text{Arctan } \frac{x+2}{2} \times 2 = \frac{1}{2} \text{Arctan } \frac{x+2}{2}$$

$$(4) \int \frac{1}{2x^2-4x+7} dx = \int \frac{1}{2(x-1)^2+5} dx = \frac{1}{5} \int \frac{1}{\left\{\sqrt{\frac{2}{5}}(x-1)\right\}^2+1} dx = \frac{1}{5} \text{Arctan } \sqrt{\frac{2}{5}}(x-1) \times \sqrt{\frac{5}{2}} = \frac{1}{\sqrt{10}} \text{Arctan } \sqrt{\frac{2}{5}}(x-1)$$

$$\text{Arcsin 型} \quad \int \frac{1}{\sqrt{1-x^2}} dx = \text{Arcsin } x$$

$$(1) \int \frac{1}{\sqrt{4-x^2}} dx = \frac{1}{2} \int \frac{1}{\sqrt{1-(\frac{x}{2})^2}} dx = \frac{1}{2} \text{Arcsin } \frac{x}{2} \times 2 = \text{Arcsin } \frac{x}{2}$$

$$(2) \int \frac{1}{\sqrt{-(x+1)^2+2}} dx = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left(\frac{x+1}{\sqrt{2}}\right)^2+1}} dx = \frac{1}{\sqrt{2}} \text{Arcsin } \frac{x+1}{\sqrt{2}} \times \sqrt{2} = \text{Arcsin } \frac{x+1}{\sqrt{2}}$$

$$(3) \int \frac{1}{\sqrt{-x^2+4x-1}} dx = \int \frac{1}{\sqrt{-(x-2)^2+3}} dx = \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{-\left(\frac{x-2}{\sqrt{3}}\right)^2+1}} dx = \frac{1}{\sqrt{3}} \text{Arcsin } \frac{x-2}{\sqrt{3}} \times \sqrt{3} = \text{Arcsin } \frac{x-2}{\sqrt{3}}$$

$$(4) \int \frac{1}{\sqrt{-2x^2+4x+1}} dx = \int \frac{1}{\sqrt{-2(x-1)^2+3}} dx = \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{-\left\{\sqrt{\frac{2}{3}}(x-1)\right\}^2+1}} dx = \frac{1}{\sqrt{3}} \text{Arcsin } \sqrt{\frac{2}{3}}(x-1) \times \sqrt{\frac{3}{2}}$$