

微分積分 II 演習問題

提出不要の演習問題です。近似式についてももう少し演習してください。頑張ってください！！

1. 次の関数について、2次近似式を求めよ。

$$(1) f(x) = \frac{1}{1-x}$$

$$(2) f(x) = \frac{1}{1-x^2}$$

$$(3) f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} (= \tanh x)$$

$$(4) f(x) = \log(1+x)$$

$$(5) f(x) = \sqrt{1+x} \text{ と } \sqrt{\frac{3}{2}} \text{ の近似値}$$

$$(6) f(x) = \sqrt[3]{1+x} \text{ と } \sqrt[3]{2} \text{ の近似値}$$

(7) $f(x) = \sqrt{\frac{1+x}{1-x}}$ と $\sqrt{\frac{3}{2}}$ の近似値

(8) $f(x) = \sqrt[3]{\frac{1+x}{1-x}}$ と $\sqrt[3]{2}$ の近似値

1. (1) $\frac{1}{1+x+x^2} = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots$ (2) $\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + \dots$ (3) $\tanh x = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \dots$ (4) $\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \frac{1}{6}x^6 + \frac{1}{7}x^7 - \frac{1}{8}x^8 + \frac{1}{9}x^9 - \frac{1}{10}x^{10} + \frac{1}{11}x^{11} - \frac{1}{12}x^{12} + \frac{1}{13}x^{13} - \frac{1}{14}x^{14} + \frac{1}{15}x^{15} - \frac{1}{16}x^{16} + \frac{1}{17}x^{17} - \frac{1}{18}x^{18} + \frac{1}{19}x^{19} - \frac{1}{20}x^{20} + \dots$ (5) $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{2048}x^5 - \frac{7}{65536}x^6 + \frac{77}{2684352}x^7 - \frac{77}{41943040}x^8 + \frac{1601}{268435200}x^9 - \frac{1601}{1048576000}x^{10} + \frac{1601}{41943040000}x^{11} - \frac{1601}{131817600000}x^{12} + \frac{1601}{4194304000000}x^{13} - \frac{1601}{13181760000000}x^{14} + \frac{1601}{419430400000000}x^{15} - \frac{1601}{1318176000000000}x^{16} + \frac{1601}{41943040000000000}x^{17} - \frac{1601}{131817600000000000}x^{18} + \frac{1601}{4194304000000000000}x^{19} - \frac{1601}{13181760000000000000}x^{20} + \dots$ (6) $\sqrt[3]{1+x} = 1 + \frac{1}{3}x - \frac{2}{9}x^2 + \frac{14}{27}x^3 - \frac{10}{27}x^4 + \frac{16}{27}x^5 - \frac{16}{27}x^6 + \frac{16}{27}x^7 - \frac{16}{27}x^8 + \frac{16}{27}x^9 - \frac{16}{27}x^{10} + \frac{16}{27}x^{11} - \frac{16}{27}x^{12} + \frac{16}{27}x^{13} - \frac{16}{27}x^{14} + \frac{16}{27}x^{15} - \frac{16}{27}x^{16} + \frac{16}{27}x^{17} - \frac{16}{27}x^{18} + \frac{16}{27}x^{19} - \frac{16}{27}x^{20} + \dots$ (7) $\sqrt{\frac{1+x}{1-x}} = \sqrt{1+x} \sqrt{\frac{1}{1-x}} = \left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{2048}x^5 - \frac{7}{65536}x^6 + \frac{77}{2684352}x^7 - \frac{77}{41943040}x^8 + \frac{1601}{268435200}x^9 - \frac{1601}{1048576000}x^{10} + \frac{1601}{41943040000}x^{11} - \frac{1601}{131817600000}x^{12} + \frac{1601}{4194304000000}x^{13} - \frac{1601}{13181760000000}x^{14} + \frac{1601}{419430400000000}x^{15} - \frac{1601}{1318176000000000}x^{16} + \frac{1601}{41943040000000000}x^{17} - \frac{1601}{131817600000000000}x^{18} + \frac{1601}{4194304000000000000}x^{19} - \frac{1601}{13181760000000000000}x^{20} + \dots\right) \left(1 + x + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{18} + \dots\right) = 1 + \frac{3}{2}x + \frac{3}{2}x^2 + \frac{9}{2}x^3 + \frac{3}{2}x^4 + \frac{27}{2}x^5 + \frac{3}{2}x^6 + \frac{81}{2}x^7 + \frac{3}{2}x^8 + \frac{243}{2}x^9 + \frac{3}{2}x^{10} + \frac{729}{2}x^{11} + \frac{3}{2}x^{12} + \frac{2187}{2}x^{13} + \frac{3}{2}x^{14} + \frac{6561}{2}x^{15} + \frac{3}{2}x^{16} + \frac{19683}{2}x^{17} + \frac{3}{2}x^{18} + \frac{59049}{2}x^{19} + \frac{3}{2}x^{20} + \dots$ (8) $\sqrt[3]{\frac{1+x}{1-x}} = \sqrt[3]{1+x} \sqrt[3]{\frac{1}{1-x}} = \left(1 + \frac{1}{3}x - \frac{2}{9}x^2 + \frac{14}{27}x^3 - \frac{10}{27}x^4 + \frac{16}{27}x^5 - \frac{10}{27}x^6 + \frac{16}{27}x^7 - \frac{10}{27}x^8 + \frac{16}{27}x^9 - \frac{10}{27}x^{10} + \frac{16}{27}x^{11} - \frac{10}{27}x^{12} + \frac{16}{27}x^{13} - \frac{10}{27}x^{14} + \frac{16}{27}x^{15} - \frac{10}{27}x^{16} + \frac{16}{27}x^{17} - \frac{10}{27}x^{18} + \frac{16}{27}x^{19} - \frac{10}{27}x^{20} + \dots\right) \left(1 + \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + \frac{16}{27}x^4 + \frac{32}{27}x^5 + \frac{64}{27}x^6 + \frac{128}{27}x^7 + \frac{256}{27}x^8 + \frac{512}{27}x^9 + \frac{1024}{27}x^{10} + \frac{2048}{27}x^{11} + \frac{4096}{27}x^{12} + \frac{8192}{27}x^{13} + \frac{16384}{27}x^{14} + \frac{32768}{27}x^{15} + \frac{65536}{27}x^{16} + \frac{131072}{27}x^{17} + \frac{262144}{27}x^{18} + \frac{524288}{27}x^{19} + \frac{1048576}{27}x^{20} + \dots\right) = 1 + \frac{5}{3}x + \frac{10}{9}x^2 + \frac{20}{27}x^3 + \frac{10}{9}x^4 + \frac{40}{27}x^5 + \frac{10}{9}x^6 + \frac{80}{27}x^7 + \frac{10}{9}x^8 + \frac{160}{27}x^9 + \frac{10}{9}x^{10} + \frac{320}{27}x^{11} + \frac{10}{9}x^{12} + \frac{640}{27}x^{13} + \frac{10}{9}x^{14} + \frac{1280}{27}x^{15} + \frac{10}{9}x^{16} + \frac{2560}{27}x^{17} + \frac{10}{9}x^{18} + \frac{5120}{27}x^{19} + \frac{10}{9}x^{20} + \dots$