

1. 次の関数の逆関数を求め、定義域と値域を述べよ。また、与えられた関数とその逆関数のグラフを描け。

(1) $y = x^2 + 2 (x \leq 0)$

値域は $y \geq 2$.

$y = x^2 + 2$

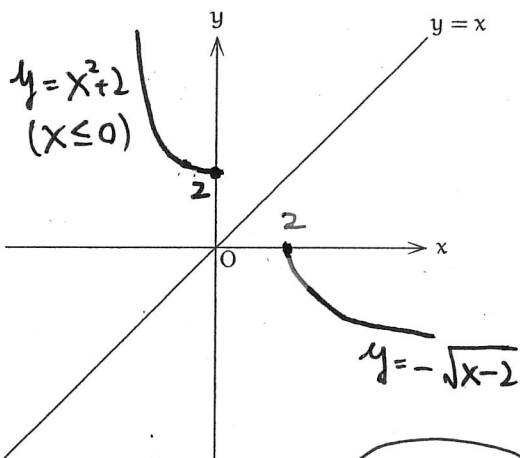
$x^2 = y - 2$

$x = \pm \sqrt{y - 2}$

$x \leq 0$ より、

$x = -\sqrt{y - 2} (x \leq 0, y \geq 2)$

$y = -\sqrt{x - 2} (x \geq 2, y \leq 0)$



(2) $y = x^2 - 4x + 5 (x \leq 2)$

$y = (x - 2)^2 + 1$

グラフより、値域は $y \geq 1$.

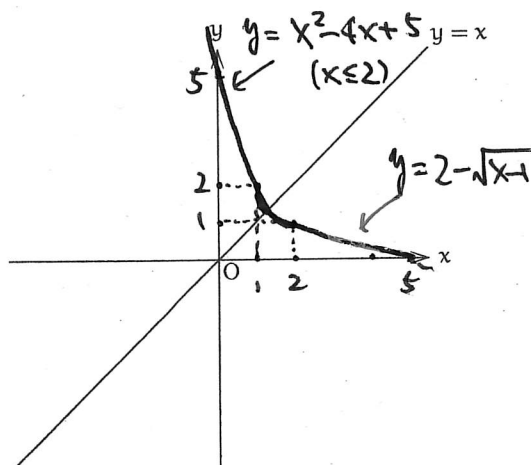
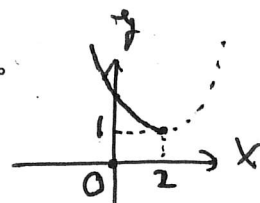
$(x - 2)^2 = y - 1$

$x - 2 = \pm \sqrt{y - 1}$

$x = 2 \pm \sqrt{y - 1}$

$x \leq 2$ より、 $x = 2 - \sqrt{y - 1} (x \leq 2, y \geq 1)$

$\therefore y = 2 - \sqrt{x - 1} (x \geq 1, y \leq 2)$



(3) $y = \frac{2}{x+2} + 2$

定義域は $x \neq -2$.

値域は $y \neq 2$.

$y = \frac{2}{x+2} + 2$

$y - 2 = \frac{2}{x+2}$

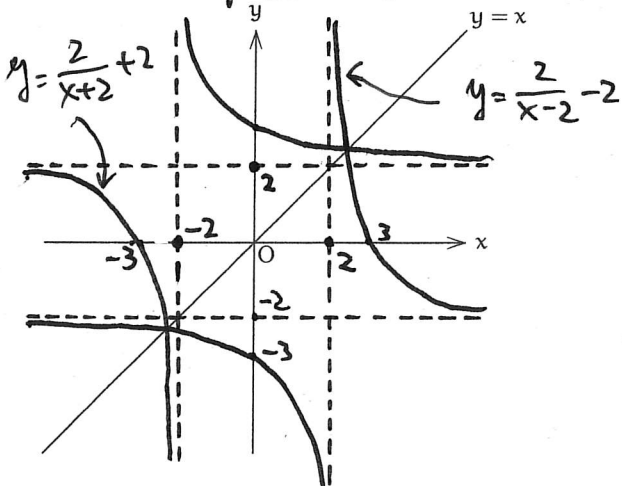
$\therefore y = \frac{2}{y-2} - 2$

$x + 2 = \frac{2}{y-2}$

$\uparrow (x \neq -2, y \neq 2)$

$x = \frac{2}{y-2} - 2 (x \neq -2, y \neq 2)$

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(4) $y = \frac{x}{x-4} = \frac{(x-4)+4}{x-4} = \frac{4}{x-4} + 1$

定義域は $x \neq 4$, 値域は $y \neq 1$.

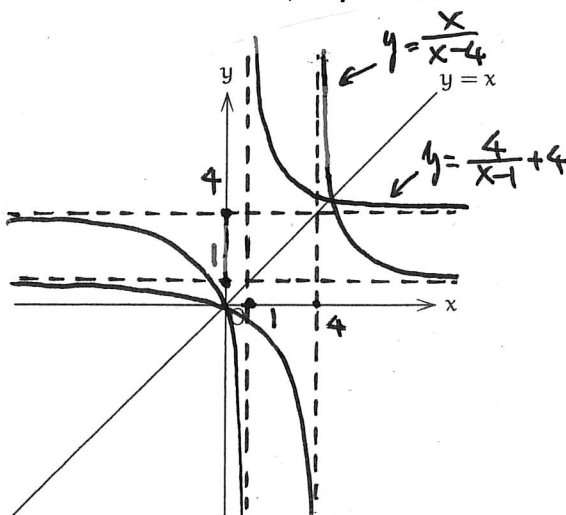
$y = \frac{4}{x-4} + 1$

$y - 1 = \frac{4}{x-4}$

$x - 4 = \frac{4}{y-1}$

$x = \frac{4}{y-1} + 4 (x \neq 4, y \neq 1)$

$y = \frac{4}{x-4} + 4 (x \neq 4, y \neq 1)$



2. $f(x) = x^2 + x$, $g(x) = -x$, $h(x) = \frac{4}{x-4} + 4$ とするとき、次の合成関数を求めよ。

(1) $(g \circ f)(x)$

$$\begin{aligned} &= g(f(x)) \\ &= g(x^2 + x) \\ &= -(x^2 + x) \\ &= -x^2 - x \end{aligned}$$

$$\begin{aligned} f(\quad) &= (\quad)^2 + (\quad) \\ g(\quad) &= -(\quad) \end{aligned}$$

と思う

(2) $(f \circ g)(x)$

$$\begin{aligned} &= f(g(x)) \\ &= f(-x) \\ &= (-x)^2 + (-x) \\ &= x^2 - x \end{aligned}$$



(3) $(g \circ h)(x)$

$$\begin{aligned} &= g(h(x)) \\ &= g\left(\frac{4}{x-4} + 4\right) \\ &= -\left(\frac{4}{x-4} + 4\right) \\ &= -\frac{4}{x-4} - 4 \end{aligned}$$

$$h(\quad) = \frac{4}{(\quad)-4} + 4$$

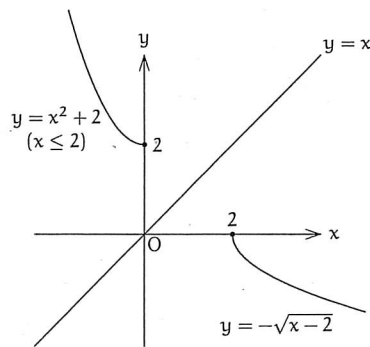
(4) $(h \circ g)(x)$

$$\begin{aligned} &= h(g(x)) \\ &= h(-x) \\ &= \frac{4}{(-x)-4} + 4 \\ &= -\frac{4}{x+4} + 4 \end{aligned}$$

(5) $(h \circ h)(x)$

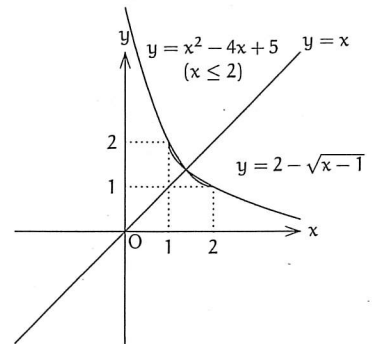
$$\begin{aligned} &= h(h(x)) \\ &= h\left(\frac{4}{x-4} + 4\right) \\ &= \frac{4}{\left(\frac{4}{x-4} + 4\right) - 4} + 4 \\ &= \frac{4}{\frac{4}{x-4}} + 4 \\ &= (x-4) + 4 \\ &= x \end{aligned}$$

(1)



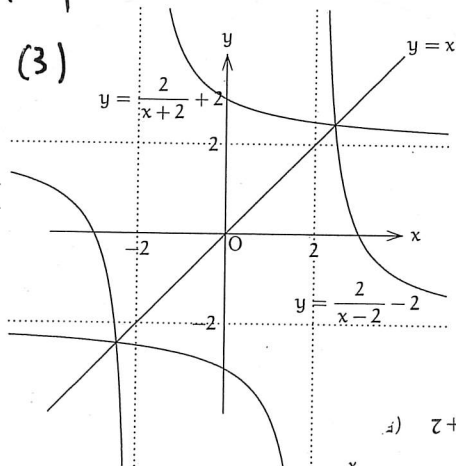
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(2)

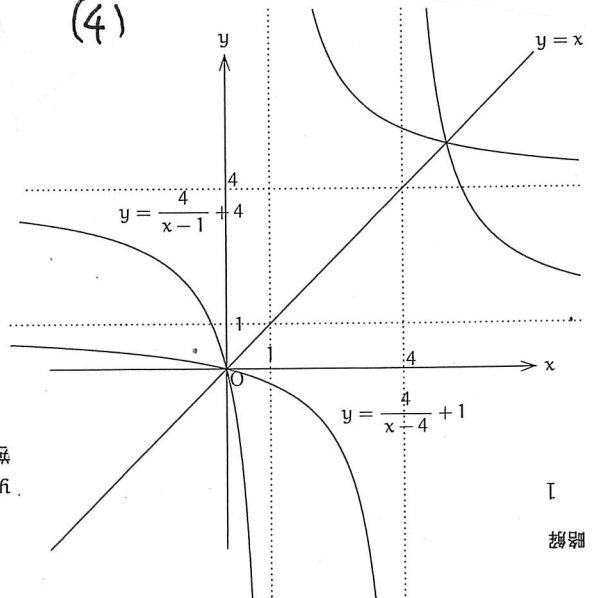


OK!

(3)



(4)



逆関数は $y = \frac{4}{x-4} + 1$ (2) 逆関数は $y = 2 - \sqrt{x-3}$ 、定義域は $x \geq 3$ 、値域は $y \leq 2$ (3) 逆関数は $y = \frac{2}{x+2} + 2$ 、定義域は $x \neq -2$ 、値域は $y \neq 4$