

基礎数学α 小課題第 21 回

裏面にある略解をもとに丸付けをすること。授業の質問も書いてくれれば回答します。名前等、忘れずにいてないに書いてください！各問題の類題もあわせて示すようにしてみました。例・例題 と 節末 は教科書の該当する章の例・例題と節末問題を、14などは問題集の番号を示しています。この課題の問題が解けなかったら教科書の例・例題に戻って確認、また、試験前には類題(例の下にある練習問題も)も解いてみると良いでしょう。

1年 ___ 科 ___ 番氏名 _____

1. $0 \leq \theta < 2\pi$ のとき、次の方程式・不等式を解け。 pp.156-157, 例題 2 (p.163), 例題 3, 節末 7, 293, 294, 299, 300

(1) $2 \cos \theta = 1$
 $\hookrightarrow \cos \theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$
 $= \frac{\pi}{3}, \frac{5}{3}\pi$

(2) $2 \sin \theta = -\sqrt{3}$
 $\hookrightarrow \sin \theta = -\frac{\sqrt{3}}{2}$

$\therefore \theta = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$
 $= \frac{5}{6}\pi, \frac{7}{6}\pi$

(3) $\tan \theta = -1$

$\therefore \theta = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$
 $= \frac{3}{4}\pi, \frac{7}{4}\pi$

(4) $2 \sin 2\theta = 1 \quad (0 \leq \theta < 2\pi)$ ^{2倍}
 (Hint: $\alpha = 2\theta$ とおくと、 $2 \sin \alpha = 1 \quad (0 \leq \alpha < 4\pi)$)

$\hookrightarrow \sin 2\theta = \frac{1}{2} \quad (0 \leq 2\theta < 4\pi)$ ← 範囲に!

$\Leftrightarrow \boxed{2\theta} = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, 3\pi - \frac{\pi}{6}$
 $2\theta = \frac{\pi}{6}, \frac{5}{6}\pi, \frac{13}{6}\pi, \frac{17}{6}\pi$
 $\therefore \theta = \frac{\pi}{12}, \frac{5}{12}\pi, \frac{13}{12}\pi, \frac{17}{12}\pi$

(5) $2 \cos(\theta - \frac{\pi}{3}) = \sqrt{3} \quad (0 \leq \theta < 2\pi)$
 (Hint: $\alpha = \theta - \frac{\pi}{3}$ とおくと、 $2 \cos \alpha = \sqrt{3} \quad (-\frac{\pi}{3} \leq \alpha < \frac{5}{3}\pi)$)

$\cos(\theta - \frac{\pi}{3}) = \frac{\sqrt{3}}{2} \quad (-\frac{\pi}{3} \leq \theta < \frac{5}{3}\pi)$

$\Leftrightarrow \boxed{\theta - \frac{\pi}{3}} = -\frac{\pi}{6}, \frac{\pi}{6}$
ここで見え

$\theta = \frac{\pi}{3} - \frac{\pi}{6}, \frac{\pi}{3} + \frac{\pi}{6}$
 $= \frac{\pi}{6}, \frac{\pi}{2}$

範囲に注意

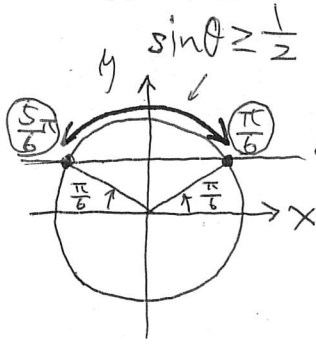
(6) $2 \cos \theta < -1 \Leftrightarrow \cos \theta < -\frac{1}{2}$

まず、 $\cos \theta = -\frac{1}{2}$ を解く。
 $\cos \theta = -\frac{1}{2}$
 $\Leftrightarrow \theta = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$
 $= \frac{2}{3}\pi, \frac{4}{3}\pi$

\square \hookrightarrow
 $\cos \theta < -\frac{1}{2}$
 $\Leftrightarrow \frac{2}{3}\pi < \theta < \frac{4}{3}\pi$

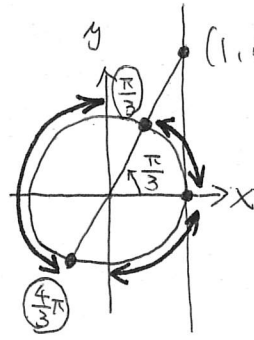
$x = -\frac{1}{2}$
 $x < -\frac{1}{2}$ は 左側

(7) $2\sin\theta \geq 1$



$\sin\theta \geq \frac{1}{2}$
 $y \geq \frac{1}{2}$ 上側
 $\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$
 $\therefore \sin\theta \geq \frac{1}{2}$
 $\Leftrightarrow \frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6}$

(8) $\tan\theta < \sqrt{3}$



$\tan\theta = \sqrt{3}$
 $\Leftrightarrow \theta = \frac{\pi}{3}, \frac{4\pi}{3}$
 $\boxtimes \neq$
 $\tan\theta < \sqrt{3}$
 $\Leftrightarrow 0 \leq \theta < \frac{\pi}{3}, \frac{\pi}{2} < \theta < \frac{4\pi}{3},$
 $\frac{3}{2} < \theta < 2\pi$

2. 加法定理を用いて、 $\sin 15^\circ, \cos 15^\circ, \tan 15^\circ$ を求めよ。例1 (p.170), 306

$$\begin{aligned} \sin 15^\circ &= \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \cos 15^\circ &= \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \tan 15^\circ &= \frac{\sin 15^\circ}{\cos 15^\circ} = \frac{\frac{\sqrt{6} - \sqrt{2}}{4}}{\frac{\sqrt{6} + \sqrt{2}}{4}} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \quad \text{有理化} \\ &= \frac{(\sqrt{6} - \sqrt{2})(\sqrt{6} - \sqrt{2})}{(\sqrt{6} + \sqrt{2})(\sqrt{6} - \sqrt{2})} \\ &= \frac{6 - 2\sqrt{12} + 2}{6 - 2} \\ &= 2 - \sqrt{3} \end{aligned}$$

加法定理

$$\left[\begin{aligned} \sin(\alpha \pm \beta) &= \sin\alpha \cos\beta \pm \cos\alpha \sin\beta \\ \cos(\alpha \pm \beta) &= \cos\alpha \cos\beta \mp \sin\alpha \sin\beta \end{aligned} \right.$$

2. $\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}, \cos 15^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}, \tan 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = 2 - \sqrt{3}$
1. (1) $\frac{\pi}{5}$ (2) $\frac{3\pi}{5}$ (3) $\frac{4\pi}{7}$ (4) $\frac{\pi}{5}, \frac{12\pi}{13}, \frac{12\pi}{17}$ (5) $\frac{\pi}{2}$ (6) $\frac{3\pi}{2} < \theta < \frac{3\pi}{4}$
- (7) $0 \leq \theta \leq \frac{6\pi}{11}$ (8) $0 \leq \theta < 2\pi$