

2年 \_\_\_\_ 科 \_\_\_\_ 番 氏名 \_\_\_\_\_

★ 不定積分を計算せよ。

$$\begin{aligned}
 (1) \int \log 2x \, dx &= \int (x)' \log 2x \, dx \\
 &= x \log 2x - \int x (\log 2x)' \, dx \\
 &= x \log 2x - \int x \cdot \frac{1}{2x} \cdot 2 \, dx \\
 &= x \log 2x - \int dx \\
 &= x \log 2x - x + C_{-1}
 \end{aligned}$$

$$\begin{aligned}
 (3) \int \frac{2x}{\sqrt{-2x+1}} \, dx & \quad t = -2x+1 \\
 &= \int \frac{1-t}{\sqrt{t}} \cdot \left(-\frac{1}{2} dt\right) \quad \begin{matrix} dt = -2dx \\ 2x = 1-t \end{matrix} \\
 &= -\frac{1}{2} \int (t^{-\frac{1}{2}} - t^{\frac{1}{2}}) dt \quad \begin{matrix} (2x \text{ も } t \text{ に直さないと} \\ \text{いけないから}) \end{matrix} \\
 &= -\frac{1}{2} \left(2t^{\frac{1}{2}} - \frac{2}{3}t^{\frac{3}{2}}\right) + C \\
 &= -t^{\frac{1}{2}} + \frac{1}{3}t^{\frac{3}{2}} + C \\
 &= -(-2x+1)^{\frac{1}{2}} + \frac{1}{3}(-2x+1)^{\frac{3}{2}} + C_{-1}
 \end{aligned}$$

$$\begin{aligned}
 (5) \int \frac{x}{x^2-1} \, dx &= \int \frac{x}{(x-1)(x+1)} \, dx \\
 \frac{x}{(x-1)(x+1)} &= \frac{a}{x-1} + \frac{b}{x+1} \quad \text{と仮定,} \\
 \text{係数比較} \quad \uparrow &= \frac{a(x+1)+b(x-1)}{(x-1)(x+1)} \\
 &= \frac{(a+b)x+(a-b)}{(x-1)(x+1)}
 \end{aligned}$$

$$\begin{cases} a+b=1 \\ a-b=0 \end{cases} \rightarrow a=\frac{1}{2}, b=\frac{1}{2}$$

$$\therefore \frac{x}{(x-1)(x+1)} = \frac{1}{2} \cdot \frac{1}{x-1} + \frac{1}{2} \cdot \frac{1}{x+1}$$

$$\begin{aligned}
 (5\text{式}) &= \int \left(\frac{1}{2} \cdot \frac{1}{x-1} + \frac{1}{2} \cdot \frac{1}{x+1}\right) dx \\
 &= \frac{1}{2} \log|x-1| + \frac{1}{2} \log|x+1| + C_{-1}
 \end{aligned}$$

$$\begin{aligned}
 (2) \int x \sin x \, dx &= \int x (-\cos x)' \, dx \\
 &= x(-\cos x) - \int (x)'(-\cos x) \, dx \\
 &= -x \cos x + \int \cos x \, dx \\
 &= -x \cos x + \sin x + C_{-1}
 \end{aligned}$$

$$\begin{aligned}
 (4) \int (2x+1)e^{-x} \, dx & \quad (e^{-x})' = -e^{-x} \text{ (合成関数の微分)} \\
 &= \int (2x+1)(-e^{-x})' \, dx \\
 &= (2x+1)(-e^{-x}) - \int (2x+1)'(-e^{-x}) \, dx \\
 &= -(2x+1)e^{-x} + 2 \int e^{-x} \, dx \\
 &= -(2x+1)e^{-x} - 2e^{-x} + C \\
 &= -(2x+3)e^{-x} + C_{-1}
 \end{aligned}$$

$$\begin{aligned}
 (6) \int \frac{x^2+3x}{x+1} \, dx & \quad \begin{matrix} -1 & 1 & 3 & 0 \\ & -1 & -2 \\ 1 & 2 & -2 \end{matrix} \\
 &= \int (x+2 - \frac{2}{x+1}) \, dx \\
 &= \frac{1}{2}x^2 + 2x - 2 \log|x+1| + C
 \end{aligned}$$