

微分積分 I 小課題第 7 回

裏面にある略解をもとに丸付けをすること。裏面も解答に使ってもよいです。授業の質問も書いてくれれば回答します。名前等、忘れずにていねいに書いてください！

2 年 ____ 科 ____ 番 氏名 _____

1. 次の極限について、収束するならば極限値を、収束しないならば、“ ∞ ”、“ $-\infty$ ”、“極限値はない”のいずれかを答えよ。

$$\begin{aligned} (1) \quad \lim_{x \rightarrow -1} \frac{x^3 + 1}{x + 1} \\ &= \lim_{x \rightarrow -1} \frac{(x+1)(x^2 - x + 1)}{x + 1} \\ &= \lim_{x \rightarrow -1} (x^2 - x + 1) \\ &= 3 \end{aligned}$$

$$\begin{aligned} (2) \quad \lim_{x \rightarrow 0} \frac{1}{x} \left(1 - \frac{1}{x+1} \right) \\ &= \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{x}{x+1} \\ &= \lim_{x \rightarrow 0} \frac{1}{x+1} \\ &= 1 \end{aligned}$$

$$\begin{aligned} (3) \quad \lim_{x \rightarrow 2} \frac{1}{x-2} \left(1 + \frac{x-7}{x^2+1} \right) \\ &= \lim_{x \rightarrow 2} \frac{1}{x-2} \cdot \frac{x^2+x-6}{x^2+1} \\ &= \lim_{x \rightarrow 2} \frac{1}{x-2} \cdot \frac{(x-2)(x+3)}{x^2+1} \\ &= \lim_{x \rightarrow 2} \frac{x+3}{x^2+1} \\ &= 1 \end{aligned}$$

$$\begin{aligned} (4) \quad \lim_{x \rightarrow 6} \frac{\sqrt{x-2}-2}{x-6} \times \frac{\sqrt{x-2}+2}{\sqrt{x-2}+2} \\ &= \lim_{x \rightarrow 6} \frac{(x-2)-4}{(x-6)(\sqrt{x-2}+2)} \\ &= \lim_{x \rightarrow 6} \frac{1}{\sqrt{x-2}+2} \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} (5) \quad \lim_{x \rightarrow 2} \frac{\sqrt{x^2+1}-\sqrt{2x+1}}{x-2} \\ &= \lim_{x \rightarrow 2} \frac{(x^2+1)-(2x+1)}{(x-2)(\sqrt{x^2+1}+\sqrt{2x+1})} \\ &= \lim_{x \rightarrow 2} \frac{x(x-2)}{(x-2)(\sqrt{x^2+1}+\sqrt{2x+1})} \\ &= \lim_{x \rightarrow 2} \frac{x}{\sqrt{x^2+1}+\sqrt{2x+1}} \\ &= \frac{1}{\sqrt{5}} \end{aligned}$$

$$\begin{aligned} (6) \quad \lim_{x \rightarrow -2} \frac{x^2-x-6}{x^2+5x+6} \\ &= \lim_{x \rightarrow -2} \frac{(x+2)(x-3)}{(x+2)(x+3)} \\ &= \lim_{x \rightarrow -2} \frac{x-3}{x+3} \\ &= -5 \end{aligned}$$

定数
↓
 $\lim_{x \rightarrow a} g(x) = 0$ かつ $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \alpha$
となるときは $\lim_{x \rightarrow a} f(x) = 0$ とな
なければならぬ

2. 次の等式が成り立つように、定数 a, b の値を定めよ。

$$\begin{aligned} (1) \quad \lim_{x \rightarrow -2} \frac{x^2+ax+b}{x+2} = -2 \\ \lim_{x \rightarrow -2} (x+2) = 0 \text{ かつ } \lim_{x \rightarrow -2} (x^2+ax+b) = 0. \\ \therefore 4-2a+b=0 \\ b=2a-4 \\ -2 = \lim_{x \rightarrow -2} \frac{x^2+ax+b}{x+2} = \lim_{x \rightarrow -2} \frac{x^2+ax+2a-4}{x+2} \\ = \lim_{x \rightarrow -2} \frac{(x+2)(x+a-2)}{x+2} \\ = \lim_{x \rightarrow -2} (x+a-2) \\ = a-4. \\ \therefore a=2 \\ b=2a-4=0. \end{aligned}$$

$$\begin{aligned} (2) \quad \lim_{x \rightarrow 1} \frac{a\sqrt{x^2+1}+b}{x-1} = 1 \\ \lim_{x \rightarrow 1} (x-1) = 0 \text{ かつ } \lim_{x \rightarrow 1} (a\sqrt{x^2+1}+b) = 0. \\ \therefore \sqrt{2}a+b=0 \\ b=-\sqrt{2}a \end{aligned}$$

$$\begin{aligned} 1 &= \lim_{x \rightarrow 1} \frac{a\sqrt{x^2+1}+b}{x-1} = \lim_{x \rightarrow 1} \frac{a\sqrt{x^2+1}-\sqrt{2}a}{x-1} \\ &= \lim_{x \rightarrow 1} a \cdot \frac{(x^2+1)-2}{(x-1)(\sqrt{x^2+1}+\sqrt{2})} \\ &= \lim_{x \rightarrow 1} a \cdot \frac{(x-1)(x+1)}{(x-1)(\sqrt{x^2+1}+\sqrt{2})} \\ &= \lim_{x \rightarrow 1} a \cdot \frac{x+1}{\sqrt{x^2+1}+\sqrt{2}} \\ &= \frac{a}{\sqrt{2}} \quad \text{よって} \end{aligned}$$

$$\begin{aligned} \therefore a &= \sqrt{2} \\ b &= -\sqrt{2}a \\ &= -2. \end{aligned}$$