

微分積分 I 小課題第 9 回

裏面にある略解をもとに丸付けをすること。裏面も解答に使ってもよいです。授業の質問も書いてくれれば回答します。名前等、忘れずにいていねいに書いてください！

2 年 ____ 科 ____ 番 氏名 _____

1. 関数 $f(x) = 2x^2 - x$ の導関数 $f'(x)$ を定義に従って求めよ。すなわち、 $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ を計算せよ。

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (\text{定義!}) &= \lim_{h \rightarrow 0} (4x + 2h - 1) \\ &= \lim_{h \rightarrow 0} \frac{2(x+h)^2 - (x+h) - \{2x^2 - x\}}{h} &= 4x - 1 \\ &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - h}{h} \end{aligned}$$

2. 次の関数を微分せよ。

(1) $f(x) = 3x^2 + 3x - 1$

$$f'(x) = 3 \cdot 2x + 3 \cdot 1 = 6x + 3$$

$$(x^r)' = r x^{r-1} \quad (r: \text{有理数})$$

$$(C)' = 0 \quad (C: \text{定数})$$

(3) $f(x) = -5x^4 + 4x^3 + 5x^2 + 2x + 2^2$

$$\begin{aligned} f'(x) &= -5 \cdot 4x^3 + 4 \cdot 3x^2 + 5 \cdot 2x + 2 \cdot 1 \\ &= -20x^3 + 12x^2 + 10x + 2 \end{aligned}$$

$$(fgh)' = f'gh + fg'h + fgh'$$

(5) $f(x) = x(x+3)(7x^2+4)$

$$\begin{aligned} f'(x) &= (x+3)(7x^2+4) + x(7x^2+4) \\ &\quad + x(x+3) \cdot 14x \end{aligned}$$

$$\begin{aligned} &= (7x^3 + 4x + 21x^2 + 28) \\ &\quad + (7x^3 + 4x) + (14x^3 + 42x^2) \\ &= 28x^3 + 63x^2 + 8x + 28 \end{aligned}$$

(2) $f(x) = 6x^5 - x^2 + 4x - 1$

$$\begin{aligned} f'(x) &= 6 \cdot 5x^4 - 2x + 4 \cdot 1 \\ &= 30x^4 - 2x + 4 \end{aligned}$$

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

(4) $f(x) = (x-2)(4x^2+x+4)$

$$\begin{aligned} f'(x) &= (x-2)'(4x^2+x+4) \\ &\quad + (x-2)(4x^2+x+4)' \\ &= (4x^2+x+4) + (x-2)(8x+1) \\ &= 12x^2 - 14x + 2 \end{aligned}$$

(6) $f(x) = \frac{x+3}{x}$

$$f'(x) = \frac{(x+3)' \cdot x - (x+3) \cdot x'}{x^2}$$

$$\begin{aligned} &= \frac{x - (x+3)}{x^2} \quad \left(\frac{f(x)'}{g(x)} \right)' \\ &= -\frac{3}{x^2} \quad = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \end{aligned}$$

今日も裏にも問題があります！

1. $f'(x) = 4x - 1$

2. (1) $f'(x) = 6x + 3$

(4) $f'(x) = 12x^2 - 14x + 2$

(10) $f'(x) = -\frac{(2x-1)(2x+1)}{4x+1}$

(7) $f'(x) = -\frac{(2x^2-1)}{4x}$

(8) $f'(x) = -\frac{(x^2-4x+1)}{2x-4}$

(5) $f'(x) = 28x^3 + 63x^2 + 8x + 12$

(6) $f'(x) = -\frac{x^2}{3}$

(9) $f'(x) = \frac{x^2-4x+2}{x^2-4x+2}$

(2) $f'(x) = 30x^4 - 2x + 4$

(3) $f'(x) = -20x^3 + 12x^2 + 10x + 2$

(8) $f(x) = \frac{1}{x^2 - 4x + 1}$

$$f'(x) = -\frac{(x^2 - 4x + 1)'}{(x^2 - 4x + 1)^2}$$

$$= -\frac{2x - 4}{(x^2 - 4x + 1)^2}$$

(7) $f(x) = \frac{1}{2x^2 - 1}$

$$f'(x) = -\frac{(2x^2 - 1)'}{(2x^2 - 1)^2}$$

忘れない!

$$= -\frac{4x}{(2x^2 - 1)^2}$$

$$\left(\frac{1}{g(x)}\right)' = -\frac{g'(x)}{g(x)^2}$$

(9) $f(x) = \frac{x^2 - 2}{x - 2}$

$$f'(x) = \frac{(x^2 - 2)' \cdot (x - 2) - (x^2 - 2) \cdot (x - 2)'}{(x - 2)^2}$$

$$= \frac{2x \cdot (x - 2) - (x^2 - 2)}{(x - 2)^2}$$

$$= \frac{x^2 - 4x + 2}{(x - 2)^2}$$

(10) $f(x) = \frac{1}{(2x - 1)(x + 1)}$

$$f'(x) = -\frac{((2x - 1)(x + 1))'}{(2x - 1)^2 (x + 1)^2}$$

$$= -\frac{2(x + 1) + (2x - 1) \cdot 1}{(2x - 1)^2 (x + 1)^2}$$

$$= -\frac{4x + 1}{(2x - 1)^2 (x + 1)^2}$$