微分積分Ⅰ　小課題第 10 回

裏面にある略解をもとに丸付けをすること。裏面も解答に使ってもよいです。授業の質問も書いてくれれば回答します。名前等、忘れずにていねいに書いてください！

2年 ＿＿ 科 ＿＿ 番　氏名 $(f(g(x)))' = f'(g(x)) \cdot g'(x)$

1. 次の関数を微分せよ。 中

(1) $f(x) = (2x+3)^3$

$f'(x) = 3(2x+3)^2 \cdot (2x+3)'$

$= 6(2x+3)^2$ (← 展開しない！)

外 $= \square^3$

外$' = 3\square^2$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

中　中の微分

外　外の微分

(2) $f(x) = (x^2+4)^8$

中の微分

$f'(x) = 8(x^2+4)^7 \cdot (x^2+4)'$

$= 16x(x^2+4)^7$

(3) $f(x) = (3-2x)^4$

$f'(x) = 4(3-2x)^3 \cdot (3-2x)'$

$= -8(3-2x)^3$

(4) $f(x) = \dfrac{1}{(2x+1)^3} = (2x+1)^{-3}$

$f'(x) = -3(2x+1)^{-4} \cdot (2x+1)'$

$= -6(2x+1)^{-4}$

$\left(= -\dfrac{6}{(2x+1)^4}\right.$

商の微分より，合成関数の微分の方が楽

(5) $f(x) = \left(\dfrac{2x-1}{x}\right)^3$

$f'(x) = 3\left(\dfrac{2x-1}{x}\right)^2 \cdot \left(\dfrac{2x-1}{x}\right)'$

$= 3\left(\dfrac{2x-1}{x}\right)^2 \cdot \dfrac{(2x-1)' \cdot x - (2x-1) \cdot (x)'}{x^2}$

$= 3\left(\dfrac{2x-1}{x}\right)^2 \cdot \dfrac{1}{x^2}$

$= \dfrac{3(2x-1)^2}{x^4}$

(6) $f(x) = (ax+b)^n$

$f'(x) = n(ax+b)^{n-1} \cdot (ax+b)'$

$= na(ax+b)^{n-1}$

(7) $f(x) = \sqrt[3]{x}$

$= x^{\frac{1}{3}}$

$f'(x) = \dfrac{1}{3}x^{\frac{1}{3}-1}$

$= \dfrac{1}{3}x^{-\frac{2}{3}}$

$\left(= \dfrac{1}{3\sqrt[3]{x^2}}\right)$

(8) $f(x) = \dfrac{1}{\sqrt[3]{x^5}} = x^{-\frac{5}{3}}$

$f'(x) = -\dfrac{5}{3}x^{-\frac{5}{3}-1}$

$= -\dfrac{5}{3}x^{-\frac{8}{3}}$

今日も裏にも問題があります！

(9) $f(x) = \sqrt{x^2+1} = (x^2+1)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot (x^2+1)'$$

$$= x(x^2+1)^{-\frac{1}{2}}$$

$$\left(= \frac{x}{\sqrt{x^2+1}} \right)$$

(10) $f(x) = \dfrac{1}{\sqrt[4]{x^2+1}} = (x^2+1)^{-\frac{1}{4}}$

$$f'(x) = -\frac{1}{4}(x^2+1)^{-\frac{5}{4}} \cdot (x^2+1)'$$

$$= -\frac{1}{2}x(x^2+1)^{-\frac{5}{4}}$$

(11) $f(x) = (x^5-1)^{\frac{7}{5}}$

$$f'(x) = \frac{7}{5}(x^5-1)^{\frac{2}{5}} \cdot (x^5-1)'$$

$$= 7x^4(x^5-1)^{\frac{2}{5}}$$

(12) $f(x) = \sqrt{\dfrac{1+x}{1-x}} = \left(\dfrac{1+x}{1-x}\right)^{\frac{1}{2}}$

中

中の微分

$$f'(x) = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \cdot \frac{(1+x)'\cdot(1-x) - (1+x)\cdot(1-x)'}{(1-x)^2}$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}} \frac{2}{(1-x)^2}$$

$$= \frac{1}{(1-x)^2}\sqrt{\frac{1-x}{1+x}}$$

(13) $f(x) = x^2 \sin x$ ← 積のビブン!

$(fg)' = f'g + fg'$

$$f'(x) = (x^2)' \sin x + x^2 (\sin x)'$$

$$= 2x \sin x + x^2 \cos x$$

(14) $f(x) = \sin x \cos x$

$$f'(x) = (\sin x)' \cos x + \sin x (\cos x)'$$

$$= \cos x \cos x + \sin x(-\sin x)$$

$$= \cos^2 x - \sin^2 x$$

$$(= \cos 2x)$$

略解

1. (1) $f'(x) = 6(2x+3)^2$ (2) $f'(x) = 16x(x^2+4)^7$ (3) $f'(x) = -8(3-2x)^3$

(4) $f'(x) = -6(2x+1)^{-4} = -\dfrac{6}{(2x+1)^4}$ (5) $f'(x) = \dfrac{3(2x-1)^2}{x^4}$ (6) $f'(x) = na(ax+b)^{n-1}$

(7) $f'(x) = \dfrac{1}{3}x^{-\frac{2}{3}} = \dfrac{1}{3\sqrt[3]{x^2}}$ (8) $f'(x) = -\dfrac{5}{3}x^{-\frac{8}{3}}$ (9) $f'(x) = x(x^2+1)^{-\frac{1}{2}} = \dfrac{x}{\sqrt{x^2+1}}$

(10) $f'(x) = -\dfrac{1}{2}x(x^2+1)^{-\frac{5}{4}}$ (11) $f'(x) = 7x^4(x^5-1)^{\frac{2}{5}}$ (12) $f'(x) = \dfrac{1}{(1-x)^2}\sqrt{\dfrac{1-x}{1+x}}$

(13) $f'(x) = 2x\sin x + x^2 \cos x$ (14) $f'(x) = \cos^2 x - \sin^2 x = \cos 2x$