微分積分Ⅰ　小課題第 11 回

裏面にある略解をもとに丸付けをすること。裏面も解答に使ってもよいです。授業の質問も書いてくれれば回答します。名前等、忘れずにていねいに書いてください！

$\{f(g(x))\}' = f'(g(x))\,g'(x)$　（外 / 外の微分 / 中 / そのまま / 中の微分）

2年 ___ 科 ___ 番　氏名 $\{f(g(x))\}' = f'(g(x))\cdot g'(x)$

1. 次の関数を微分せよ。

(1) $f(x) = \sin(2x+3)$

$f'(x) = \cos(2x+3)\cdot(2x+3)'$

$= 2\cos(2x+3)$

$\cos 2(2x+3)$ ではない。

(2) $f(x) = \cos^2 x = (\cos x)^2$

$f'(x) = 2\cos x\cdot(\cos x)'$

$= -2\sin x\cos x$

$(\tan\square)' = \dfrac{1}{\cos^2\square}$

(3) $f(x) = \tan(4x+5)$

$f'(x) = \dfrac{1}{\cos^2(4x+5)}\cdot(4x+5)'$

$= \dfrac{4}{\cos^2(4x+5)}$

(4) $f(x) = \sin^3(1-x) = (\sin(1-x))^3$

$f'(x) = 3(\sin(1-x))^2\cdot(\sin(1-x))'$

$= 3\sin^2(1-x)\cdot\cos(1-x)\cdot(1-x)'$

$= -3\sin^2(1-x)\cos(1-x)$

※ $(\sin(1-x))' = \cos(1-x)\cdot(1-x)'$

$= -\cos(1-x)$

(5) $f(x) = \cos^2 3x = (\cos 3x)^2$

$f'(x) = 2\cos 3x\cdot(\cos 3x)'$

$= 2\cos 3x\cdot(-\sin 3x)\cdot(3x)'$

$= -6\sin 3x\cos 3x$

$(= -3\sin 6x)$

(6) $f(x) = \dfrac{1}{\tan 2x}$

$= (\tan 2x)^{-1}$

$f'(x) = -(\tan 2x)^{-2}\cdot(\tan 2x)'$

$= -\dfrac{1}{\tan^2 2x}\cdot\dfrac{1}{\cos^2 2x}\cdot(2x)'$

$= -\dfrac{2}{\cos^2 2x\tan^2 2x}$

(7) $f(x) = \sin(\pi\sin x)$

$f'(x) = \cos(\pi\sin x)\cdot(\pi\sin x)'$

$= \cos(\pi\sin x)\cdot\pi\cos x$

$= \pi\cos x\cos(\pi\sin x)$

(8) $f(x) = \mathrm{Arcsin}\,x^2$

$f'(x) = \dfrac{1}{\sqrt{1-(x^2)^2}}\cdot(x^2)'$

$= \dfrac{2x}{\sqrt{1-x^4}}$

今日も裏にも問題があります！

(9) $f(x) = \operatorname{Arccos}\sqrt{x}$

$$f'(x) = -\frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot (\sqrt{x})'$$
$$= -\frac{1}{\sqrt{1-x}} \cdot \frac{1}{2} x^{-\frac{1}{2}}$$
$$= -\frac{1}{2\sqrt{x(1-x)}}$$

(10) $f(x) = \operatorname{Arctan}\sqrt{\frac{1+x}{1-x}}$

$$f'(x) = \frac{1}{1+\left(\sqrt{\frac{1+x}{1-x}}\right)^2} \cdot \left(\sqrt{\frac{1+x}{1-x}}\right)'$$
$$= \frac{1}{1+\frac{1+x}{1-x}} \cdot \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \cdot \left(\frac{1+x}{1-x}\right)'$$
$$= \frac{1-x}{(1-x)+(1+x)} \cdot \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}} \cdot \frac{1\cdot(1-x)-(1+x)\cdot(-1)}{(1-x)^2}$$
$$= \frac{1}{2(1-x)}\sqrt{\frac{1-x}{1+x}}$$

(11) $f(x) = \log(x^2+x)$

$$f'(x) = \frac{1}{x^2+x} \cdot (x^2+x)'$$
$$= \frac{2x+1}{x^2+x}$$

(12) $f(x) = \log_3(x^2+1)$

$$f'(x) = \frac{1}{(x^2+1)\log 3} \cdot (x^2+1)'$$
$$= \frac{2x}{(x^2+1)\log 3}$$

$(\log_a \square)' = \frac{1}{\square \log a}$

(13) $f(x) = \log|x^3-x|$

$$f'(x) = \frac{1}{x^3-x} \cdot (x^3-x)'$$
$$= \frac{3x^2-1}{x^3-x}$$

$(\log|\square|)' = \frac{1}{\square}$

(14) $f(x) = \log|\tan x|$

$$f'(x) = \frac{1}{\tan x} \cdot (\tan x)'$$
$$= \frac{1}{\tan x} \cdot \frac{1}{\cos^2 x}$$
$$= \frac{1}{\sin x \cos x}$$

略解

1. (1) $f'(x) = 2\cos(2x+3)$
(2) $f'(x) = -2\sin x\cos x = -\sin 2x$
(3) $f'(x) = \frac{4}{\cos^2(4x+5)} = 4(1+\tan^2(4x+5))$
(4) $f'(x) = -3\sin^2(1-x)\cos(1-x)$
(5) $f'(x) = -6\sin 3x\cos 3x = -3\sin 6x$
(6) $f'(x) = -\frac{2}{\cos^2 2x\tan^2 2x} = -\frac{2}{\sin^2 2x}$
(7) $f'(x) = \pi\cos(\pi\sin x)\cos x$
(8) $f'(x) = \frac{2x}{\sqrt{1-x^4}}$
(9) $f'(x) = -\frac{1}{2\sqrt{x}(1-x)}$
(10) $f'(x) = \frac{1}{2(1-x)}\sqrt{\frac{1-x}{1+x}} = \frac{1}{2\sqrt{1-x^2}}$
(11) $f'(x) = \frac{2x+1}{x^2+x}$
(12) $f'(x) = \frac{2x}{(x^2+1)\log 3}$
(13) $f'(x) = \frac{3x-1}{x^3-x}$
(14) $f'(x) = \frac{1}{\cos^2 x\tan x} = \frac{1}{\sin x\cos x}$