微分積分 I　小課題第 16 回

裏面にある略解をもとに丸付けをすること。裏面も解答に使ってもよいです。授業の質問も書いてくれれば回答します。名前等、忘れずにていねいに書いてください！

2年 ＿＿ 科 ＿＿ 番　氏名 ＿＿＿＿＿＿＿＿＿＿

1. 次の不定積分を計算せよ。

(1) $\int e^{x+1}dx$

$= e^{x+1} + C$

$(e^{x+1})' = e^{x+1}$

$\Leftrightarrow e^{x+1} + C = \int e^{x+1}dx$

(2) $\int 2^{2x+1}dx$

$t = 2x+1$ とおくと，

$dt = \frac{dt}{dx}dx = 2dx$

$= \int 2^t \cdot \frac{1}{2}dt$

$= \frac{1}{2}\cdot\frac{2^t}{\log 2} + C = \frac{1}{2}\cdot\frac{2^{2x+1}}{\log 2} + C = \frac{2^{2x}}{\log 2} + C$

(3) $\int \cos(5x+1)dx$

$t = 5x+1$ とおくと

$dt = 5dx$

$= \int \cos t \cdot \frac{1}{5}dt$

$= \frac{1}{5}\sin t + C$

xに直す

$= \frac{1}{5}\sin(5x+1) + C$

(4) $\int (5x-1)^4 dx$

$t = 5x-1$ とおくと

$dt = 5dx$

$= \int t^4 \cdot \frac{1}{5}dt$

$= \frac{1}{5}\cdot\frac{1}{5}t^5 + C$

$= \frac{1}{25}(5x-1)^5 + C$

(5) $\int \sqrt{3x+10}\,dx$

$t = 3x+10$

$dt = 3dx$

$= \int t^{\frac{1}{2}} \cdot \frac{1}{3}dt$

$= \frac{1}{3}\cdot\frac{2}{3}t^{\frac{3}{2}} + C$

$= \frac{2}{9}(3x+10)^{\frac{3}{2}} + C$

(6) $\int (2x^2 - x + 1)^3(4x-1)dx$

$t = 2x^2 - x + 1$

$dt = (4x-1)dx$

$= \int t^3 dt$

$= \frac{1}{4}t^4 + C$

$= \frac{1}{4}(2x^2 - x + 1)^4 + C$

(7) $\int \frac{4x}{\sqrt{2x-1}}dx$

$t = 2x-1$ とおくと，

$x = \frac{1}{2}(t+1)$

$dx = \frac{dx}{dt}dt = \frac{1}{2}dt$

$= \int \frac{2(t+1)}{\sqrt{t}} \cdot \frac{1}{2}dt$

$= \int (t^{\frac{1}{2}} + t^{-\frac{1}{2}})dt$

$= \frac{2}{3}t^{\frac{3}{2}} + 2t^{\frac{1}{2}} + C$

$= \frac{2}{3}(2x-1)^{\frac{3}{2}} + 2(2x-1)^{\frac{1}{2}} + C$

(8) $\int x\sqrt[3]{-x+1}\,dx$

$t = -x+1$ とおく．

$x = -t+1$

$dx = -dt$

$= \int (-t+1)t^{\frac{1}{3}} \cdot (-dt)$

$= \int (t^{\frac{4}{3}} - t^{\frac{1}{3}})dt$

$= \frac{3}{7}t^{\frac{7}{3}} - \frac{3}{4}t^{\frac{4}{3}} + C$

$= \frac{3}{7}(-x+1)^{\frac{7}{3}} - \frac{3}{4}(-x+1)^{\frac{4}{3}} + C$

今日は裏にも問題があります！

(9) $\int (6x+1)\sqrt{3x-2}\,dx$

$t=3x-2$ とおくと、

$x=\frac{1}{3}(t+2)$

$dx=\frac{1}{3}dt$

$= \int (2t+5)\sqrt{t}\cdot\frac{1}{3}dt$

$= \frac{1}{3}\int (2t^{\frac{3}{2}}+5t^{\frac{1}{2}})dt$

$= \frac{1}{3}\left(2\cdot\frac{2}{5}t^{\frac{5}{2}}+5\cdot\frac{2}{3}t^{\frac{3}{2}}\right)+C$

$= \frac{4}{15}(3x-2)^{\frac{5}{2}}+\frac{10}{9}(3x-2)^{\frac{3}{2}}+C$

(10) $\int \sin^7 x\cos x\,dx$

$t=\sin x$ とおくと

$dt=\frac{dt}{dx}dx=\cos x\,dx$

$= \int t^7\cdot dt$

$= \frac{1}{8}t^8+C$

$= \frac{1}{8}\sin^8 x+C$

(11) $\int \frac{\tan^2 x}{\cos^2 x}dx$

$t=\tan x$ とおくと、

$dt=\frac{1}{\cos^2 x}dx$

$= \int t^2 dt$

$= \frac{1}{3}t^3+C$

$= \frac{1}{3}\tan^3 x+C$

(12) $\int \frac{e^{2x}}{e^x+e^{-x}}dx$

$t=e^x$ とおくと、

$dt=e^x dx$

$= t\,dx$

$= \int \frac{t^2}{t+\frac{1}{t}}\cdot\frac{1}{t}dt$

$= \int \frac{t^2}{t^2+1}dt$

$= \int \left(1-\frac{1}{1+t^2}\right)dt$

$\left((\mathrm{Arctan}\,x)' = \frac{1}{1+x^2} \right)$

$= t-\mathrm{Arctan}\,t+C$

$= e^x-\mathrm{Arctan}\,e^x+C$

略解

1. (1) $e^{x+1}+C$ (2) $\frac{2^{2x}}{\log 2}+C$ (3) $\frac{1}{5}\sin(5x+1)+C$ (4) $\frac{1}{25}(5x-1)^5+C$

(5) $\frac{2}{9}(3x+10)^{\frac{3}{2}}+C$ (6) $\frac{1}{4}(2x^2-x+1)^4+C$ (7) $\frac{2}{3}(2x-1)^{\frac{3}{2}}+2(2x-1)^{\frac{1}{2}}+C$

(8) $\frac{3}{7}(-x+1)^{\frac{7}{3}}-\frac{3}{4}(-x+1)^{\frac{4}{3}}+C$ (9) $\frac{4}{15}(3x-2)^{\frac{5}{2}}+\frac{10}{3}(3x-2)^{\frac{3}{2}}+C$

(10) $\frac{1}{8}\sin^8 x+C$ (11) $\frac{1}{3}\tan^3 x+C$ (12) $e^x-\mathrm{Arctan}\,e^x+C$