微分積分Ⅰ　小課題第 18 回

裏面にある略解をもとに丸付けをすること。裏面も解答に使ってもよいです。授業の質問も書いてくれれば回答します。名前等、忘れずにていねいに書いてください！

2年 ___ 科 ___ 番　氏名 _____

1. 次の定積分を計算せよ。

(1) $\int_1^8 \sqrt[3]{x}\,dx$

$= \int_1^8 x^{\frac{1}{3}}\,dx$

$= \left[\frac{3}{4}x^{\frac{4}{3}}\right]_1^8$

$= \frac{3}{4}\left(8^{\frac{4}{3}} - 1\right)$

$= \frac{45}{4}$

$8^{\frac{4}{3}} = \left(2^3\right)^{\frac{4}{3}}$

$= 2^{3\times\frac{4}{3}}$

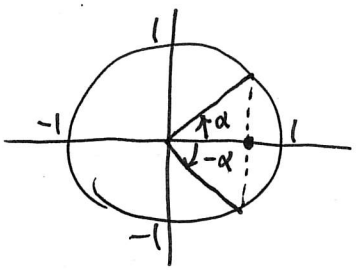
$= 2^4$

(2) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin x\,dx = \left[-\cos x\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$= -\left(\cos\frac{\pi}{4} - \cos\left(-\frac{\pi}{4}\right)\right)$

$= 0$

$\cos(-\alpha) = \cos\alpha$



(3) $\int_0^{\frac{\pi}{6}} \cos 2x\,dx$

$= \left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\left(\sin\frac{\pi}{3} - \sin 0\right)$

$= \frac{1}{2}\cdot\left(\frac{\sqrt{3}}{2} - 0\right)$

$= \frac{\sqrt{3}}{4}$

(4) $\int_{-1}^{1} e^{x+1}\,dx$

$= \left[e^{x+1}\right]_{-1}^{1}$

$= e^2 - e^0$

$= e^2 - 1$

$a^{\log_a M} = M$

(5) $\int_e^{2e} \frac{1}{2x}\,dx$

$= \frac{1}{2}\int_e^{2e} \frac{1}{x}\,dx$

$= \frac{1}{2}\left[\log|x|\right]_e^{2e}$

$= \frac{1}{2}(\log 2e - \log e)$

$= \frac{1}{2}\log 2$

$\log M - \log N = \log\frac{M}{N}$

(6) $\int_1^{2\log_3 2} 3^x\,dx$

$= \left[\frac{3^x}{\log 3}\right]_1^{2\log_3 2}$

$= \frac{1}{\log 3}\left(3^{2\log_3 2} - 3^1\right)$

$= \frac{1}{\log 3}\left(3^{\log_3 2^2} - 3\right)$

$= \frac{1}{\log 3}\left(2^2 - 3\right)$

$= \frac{1}{\log 3}$

$r\log_a M = \log_a M^r$

今日は裏にも問題があります！

$\int_{\text{始}}^{\text{終}} f(x)\,dx$

(7) $\int_1^2 (x-1)\sqrt{2-x}\,dx$

$t = 2-x$

$dt = -dx$

また、$x = 2-t$

$x-1 = 1-t$

x	$1 \to 2$
t	$1 \to 0$

$x=1$ のとき、$t = 2-x = 2-1 = 1$

$x=2$ のときの t の値

$= \int_1^0 (1-t)\sqrt{t}\cdot(-dt)$ （$(1-t)$ に -1 がかける）

$= -\int_1^0 (t^{\frac{1}{2}} - t^{\frac{3}{2}})\,dt$

$\int_a^b = -\int_b^a$

$= \int_0^1 (t^{\frac{1}{2}} - t^{\frac{3}{2}})\,dt$

$= \left[\frac{2}{3}t^{\frac{3}{2}} - \frac{2}{5}t^{\frac{5}{2}}\right]_0^1$

$= \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$

(8) $\int_{-\frac{1}{2}}^{\frac{1}{2}} (2x+3)^3\,dx$

$t = 2x+3$

$dt = 2dx$

x	$-\frac{1}{2} \to \frac{1}{2}$
t	$2 \to 4$

$= \int_2^4 t^3 \cdot \frac{1}{2}\,dt$

$= \frac{1}{2}\left[\frac{1}{4}t^4\right]_2^4$

$= \frac{1}{8}(4^4 - 2^4)$

$= \frac{1}{8}\cdot 2^4(2^4-1)$

$= 2(16-1) = 30$

(9) $\int_{-\pi}^{\pi} \sin^4 x \cos x\,dx$

$t = \sin x$

$dt = \cos x\,dx$

x	$-\pi \to \pi$
t	$0 \to 0$

$= \int_0^0 t^4\,dt$

$= 0$

$\int_a^a f(x)\,dx = 0$

(10) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\tan x}\,dx$

log 型ごまかできる！

$t = \sin x$

$dt = \cos x\,dx$

x	$\frac{\pi}{4} \to \frac{\pi}{3}$
t	$\frac{1}{\sqrt{2}} \to \frac{\sqrt{3}}{2}$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos x}{\sin x}\,dx$

$= \int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{1}{t}\,dt$

$= \left[\log|t|\right]_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}}$

$= \log\frac{\sqrt{3}}{2} - \log\frac{1}{\sqrt{2}}$

$= \log\left(\frac{\sqrt{3}}{2} \times \sqrt{2}\right)$

$= \log\frac{\sqrt{6}}{2}$

略解

1. (1) $\left[\frac{3}{4}x^{\frac{4}{3}}\right]_1^8 = \frac{45}{4}$ (2) $\left[-\cos x\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = 0$ (3) $\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{6}} = \frac{\sqrt{3}}{4}$

(4) $\left[e^{x+1}\right]_{-1}^{1} = e^2 - 1$ (5) $\frac{1}{2}\left[\log x\right]_e^{2e} = \frac{1}{2}\log 2$ (6) $\left[\frac{3^x}{\log 3}\right]_1^{2\log_3 2} = \frac{1}{\log 3}$

(7) $\left[-\frac{2}{5}t^{\frac{5}{2}} + \frac{2}{3}t^{\frac{3}{2}}\right]_0^1 = \frac{4}{15}$ $t = 2-x$, $dt = -dx$

(8) $\left[\frac{1}{4}t^4 \times \frac{1}{2}\right]_2^4 = 30$ $t = 2x+3$, $dt = 2dx$

(9) $\left[t^2\right]_0^0 = 0$ $t = \sin x$, $dt = \cos x\,dx$

(10) $\left[\frac{1}{2}\log|t|\right]_{\frac{1}{\sqrt{2}}}^{\frac{1}{2}} = -\frac{1}{2}\log 2$ $t = \sin 2x$, $dt = 2\cos 2x\,dx$