ひと目の微分

- ▷ 微分計算の力を付ける＋α の問題集です。典型的な微分を取り上げてみました。微分計算の呼吸を身につけましょう。
- ▷ 最初は簡単な問題をたくさんこなすのが、微分上達のコツかと思います。どの難しそうな微分も簡単なものの組み合わせから成っています。
- ▷ 以下の関数の微分をひと目で解いてください。
- ▷ 関数をひと目見て、計算の仕方がわかるものは計算する必要がないと思います。
- ▷ ひと目見てわからない問題は、ノートに書いて計算してみましょう。
- ▷ 考えてわからない問題は、解答を一読したら、解答を伏せてもう一度。

基礎編～覚える微分公式～

□ x^3　□ $\sqrt{x}$　□ $\dfrac{1}{x^2}$　□ $\dfrac{1}{\sqrt[3]{x^2}}$　□ $\sin x$

□ $\cos x$　□ $\tan x$　□ $\mathrm{Arcsin}\, x$　□ $\mathrm{Arccos}\, x$　□ $\mathrm{Arctan}\, x$

□ $\log x$　□ $\log|x|$　□ $\log_a x$　□ e^x　□ 3^x

積の微分 $(fg)' = f'g + fg'$

□ $x^2 \cos x$　□ $x \log x$　□ $x^2 e^x$　□ $\dfrac{\log x}{x^2}$　□ $x^n e^{-x}$

商の微分 $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$, $\left(\dfrac{1}{g}\right)' = -\dfrac{g'}{g^2}$

□ $\dfrac{1}{x^2+1}$　□ $\dfrac{1}{(3x+1)^4}$　□ $\dfrac{x^2}{x^2+1}$　□ $\dfrac{1-x}{1+x}$　□ $\dfrac{1}{\tan x}$

□ $\dfrac{1+\sin x}{1-\sin x}$　□ $\dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

合成関数の微分 $\big(f(g(x))\big)' = f'(g(x)) \cdot (g(x))'$

□ $(2x^2+1)^3$　□ $\dfrac{1}{(3x+1)^4}$　□ $\dfrac{1}{\sqrt[5]{5x-3}}$　□ $\left(x - \dfrac{1}{x}\right)^4$　□ $\cos 3x$

□ $\sin(x^2+1)$　□ $\sin^2 x$　□ $\dfrac{1}{\cos 2x}$　□ $\sin(\sin x)$　□ $\mathrm{Arccos}\, 2x$

□ $\mathrm{Arcsin}\sqrt{x}$　□ $\mathrm{Arctan}\sqrt{x}$　□ $\log(2x+1)$　□ $\log|1-3x|$　□ $\log|\cos x|$

□ $(\log_a x)^3$　□ e^{-3x+1}　□ e^{x^2}　□ $10^{\sin x}$　□ $\sqrt{\dfrac{1+x}{1-x}}$

□ $\sin x \cos^2 x$　□ $\tan^2(3x+5)$

対数微分法 $(\log|f(x)|)' = \dfrac{f'(x)}{f(x)}$

□ $\sqrt{\dfrac{1+x}{1-x}}$　□ x^x

基礎編

☐ $(x^3)' = 3x^2$ ☐ $(\sqrt{x})' = \frac{1}{2}x^{-\frac{1}{2}}$ ☐ $\left(\frac{1}{x^2}\right)' = -2x^{-3}$ ☐ $\left(\frac{1}{\sqrt[3]{x^2}}\right)' = -\frac{2}{3}x^{-\frac{5}{3}}$ ☐ $(\sin x)' = \cos x$

☐ $(\cos x)' = -\sin x$ ☐ $(\tan x)' = \frac{1}{\cos^2 x}$ ☐ $(\mathrm{Arcsin}\, x)' = \frac{1}{\sqrt{1-x^2}}$ ☐ $(\mathrm{Arccos}\, x)' = -\frac{1}{\sqrt{1-x^2}}$ ☐ $(\mathrm{Arctan}\, x)' = \frac{1}{1+x^2}$

☐ $(\log x)' = \frac{1}{x}$ ☐ $(\log|x|)' = \frac{1}{x}$ ☐ $(\log_a x)' = \frac{1}{x\log a}$ ☐ $(e^x)' = e^x$ ☐ $(3^x)' = 3^x \log 3$

積の微分 $(fg)' = f'g + fg'$

☐ $(x^2\cos x)' = (x^2)'\cos x + x^2(\cos x)' = 2x\cos x - x^2\sin x$

☐ $(x\log x)' = (x)'\log x + x(\log x)' = \log x + 1$

☐ $(x^2e^x)' = (x^2)'e^x + x^2(e^x)' = 2xe^x + x^2e^x$

☐ $\left(\frac{\log x}{x^2}\right)' = (x^{-2}\log x)' = -2x^{-3}\cdot\log x + x^{-2}\cdot\frac{1}{x}$
$= -2x^{-3}\log x + x^{-3}$

☐ $(x^ne^{-x})' = (x^n)'e^{-x} + x^n(e^{-x})' = nx^{n-1}e^{-x} - x^ne^{-x}$

商の微分 $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$, $\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$

☐ $\left(\frac{1}{x^2+1}\right)' = -\frac{(x^2+1)'}{(x^2+1)^2} = -\frac{2x}{(x^2+1)^2}$

☐ $\left(\frac{1}{(3x+1)^4}\right)' = -\frac{((3x+1)^4)'}{((3x+1)^4)^2} = -\frac{12(3x+1)^3}{(3x+1)^8}$
$= -\frac{12}{(3x+1)^5}$

☐ $\left(\frac{x^2}{x^2+1}\right)' = \frac{(x^2)'\cdot(x^2+1) - x^2\cdot(x^2+1)'}{(x^2+1)^2}$
$= \frac{2x}{(x^2+1)^2}$

☐ $\left(\frac{1-x}{1+x}\right)' = \frac{(1-x)'\cdot(1+x) - (1-x)\cdot(1+x)'}{(1+x)^2}$
$= -\frac{2}{(x^2+1)^2}$

☐ $\left(\frac{1}{\tan x}\right)' = -\frac{(\tan x)'}{\tan^2 x} = -\frac{1}{\cos^2 x\tan^2 x}\ \left(= -\frac{1}{\sin^2 x}\right)$

☐ $\frac{1+\sin x}{1-\sin x} = \frac{(1+\sin x)'\cdot(1-\sin x) - (1+\sin x)\cdot(1-\sin x)'}{(1-\sin x)^2}$
$= \frac{2\cos x}{(1-\sin x)^2}$

☐ $\frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{(e^x - e^{-x})'\cdot(e^x + e^{-x}) - (e^x - e^{-x})\cdot(e^x + e^{-x})'}{(e^x + e^{-x})^2}$
$= \frac{4}{(e^x + e^{-x})^2}$

合成関数の微分 $(f(g(x)))' = f'(g(x))\cdot(g(x))'$

☐ $((2x^2+1)^3)' = 3(2x^2+1)^2\cdot(2x^2+1)' = 12x(2x^2+1)^2$

☐ $\left(\frac{1}{(3x+1)^4}\right)' = ((3x+1)^{-4})' = -4(3x+1)^{-5}\cdot(3x+1)'$
$= -12(3x+1)^{-5}$

☐ $\left(\frac{1}{\sqrt[5]{5x-3}}\right)' = ((5x-3)^{-\frac{1}{5}})' = -\frac{1}{5}(5x-3)^{-\frac{6}{5}}\cdot(5x-3)'$
$= -(5x-3)^{-\frac{6}{5}}$

☐ $\left(\left(x-\frac{1}{x}\right)^4\right)' = 4\left(x-\frac{1}{x}\right)^3\cdot\left(x-\frac{1}{x}\right)'$
$= 4\left(x-\frac{1}{x}\right)^3\left(1+\frac{1}{x^2}\right)$

☐ $(\cos 3x)' = -\sin 3x\cdot(3x)' = -3\sin 3x$

☐ $(\sin(x^2+1))' = \cos(x^2+1)\cdot(x^2+1)' = 2x\cos(x^2+1)$

☐ $(\sin^2 x)' = ((\sin x)^2)' = 2\sin x\cdot(\sin x)' = 2\sin x\cos x$

☐ $\left(\frac{1}{\cos 2x}\right)' = ((\cos 2x)^{-1})' = -(\cos 2x)^{-2}\cdot(\cos 2x)'$
$= \frac{2\sin 2x}{\cos^2 2x}$

☐ $(\sin(\sin x))' = \cos(\sin x)\cdot(\sin x)' = \cos x\cos(\sin x)$

☐ $(\mathrm{Arccos}\, 2x)' = -\frac{1}{\sqrt{1-(2x)^2}}\cdot(2x)' = -\frac{2}{\sqrt{1-4x^2}}$

☐ $(\mathrm{Arcsin}\sqrt{x})' = \frac{1}{\sqrt{1-(\sqrt{x})^2}}\cdot(\sqrt{x})' = \frac{1}{\sqrt{1-x}}\cdot\frac{1}{2}x^{-\frac{1}{2}}$
$= \frac{1}{2\sqrt{x(1-x)}}$

☐ $(\mathrm{Arctan}\sqrt{x})' = \frac{1}{1+(\sqrt{x})^2}\cdot(\sqrt{x})' = \frac{1}{1+x}\cdot\frac{1}{2}x^{-\frac{1}{2}}$
$= \frac{1}{2(1+x)\sqrt{x}}$

☐ $(\log(2x+1))' = \frac{1}{2x+1}\cdot(2x+1)' = \frac{2}{2x+1}$

☐ $(\log|1-3x|)' = \frac{1}{1-3x}\cdot(1-3x)' = \frac{3}{3x-1}$

☐ $(\log|\cos x|)' = \frac{1}{\cos x}\cdot(\cos x)' = -\frac{\sin x}{\cos x}\ (= -\tan x)$

☐ $((\log_a x)^3)' = 3(\log_a x)^2\cdot(\log_a x)' = \frac{3(\log_a x)^2}{x\log a}$

☐ $(e^{-3x+1})' = e^{-3x+1}\cdot(-3x+1)' = -3e^{-3x+1}$

☐ $(e^{x^2})' = e^{x^2}\cdot(x^2)' = 2xe^{x^2}$

☐ $(10^{\sin x})' = 10^{\sin x}\log 10\cdot(\sin x)' = -10^{\sin x}\log 10\cos x$

☐ $\left(\sqrt{\frac{1+x}{1-x}}\right)' = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\cdot\left(\frac{1+x}{1-x}\right)' = \frac{1}{2}\sqrt{\frac{1-x}{1+x}}\cdot\frac{1\cdot(1-x)-(1+x)\cdot(-1)}{(1-x)^2} = \frac{1}{(1-x)^2}\sqrt{\frac{1-x}{1+x}}$

☐ $(\sin x\cos^2 x)' = (\sin x)'\cdot\cos^2 x + \sin x\cdot((\cos x)^2)' = \cos x\cdot\cos^2 x + \sin x\cdot 2\cos x\cdot(\cos x)' = \cos^3 x - 2\sin^2 x\cos x$

☐ $(\tan^2(3x+5))' = ((\tan(3x+5))^2)' = 2\tan(3x+5)\cdot(\tan(3x+5))' = 2\tan(3x+5)\cdot\frac{1}{\cos^2(3x+5)}\cdot(3x+5)' = \frac{6\tan(3x+5)}{\cos^2(3x+5)}$

対数微分法 $(\log|f(x)|)' = \frac{f'(x)}{f(x)}$

☐ $f(x) = \sqrt{\frac{1+x}{1-x}}$ に対し、自然対数を取る:

$$\log|f(x)| = \log\left|\sqrt{\frac{1+x}{1-x}}\right| = \frac{1}{2}\log\left|\frac{1+x}{1-x}\right| = \frac{1}{2}\log|1+x| - \frac{1}{2}\log|1-x|$$

両辺を微分:

$$\frac{f'(x)}{f(x)} = \frac{1}{2}\cdot\frac{1}{1+x}\cdot(1+x)' - \frac{1}{2}\cdot\frac{1}{1-x}\cdot(1-x)' = \frac{1}{(1+x)(1-x)}$$

$$\therefore\quad f'(x) = \frac{1}{(1+x)(1-x)}\cdot f(x) = \frac{1}{(1+x)(1-x)}\sqrt{\frac{1+x}{1-x}}$$

☐ $f(x) = x^x\ (x>0)$ に対し、自然対数を取る:

$$\log f(x) = \log x^x = x\log x$$

両辺を微分:

$$\frac{f'(x)}{f(x)} = (x)'\cdot\log x + x\cdot(\log x)' = \log x + x\cdot\frac{1}{x} = \log x + 1$$

$$\therefore\quad f'(x) = (1+\log x)\cdot f(x) = (1+\log x)\,x^x$$