ひと目の積分

- ▷ 積分計算の基礎力を付ける＋α の問題集です。典型的な問題を取り上げてみました。
- ▷ まずは、微分の問題をひと目で解いてください。微分ができなければ積分はできません。
- ▷ 積分の問題をひと目見て、解き方がわかるものは計算する必要がないと思います。
- ▷ ひと目見てわからない問題は、ノートに書いて計算してみましょう。
- ▷ 考えてわからない問題は解答を見て、読んだら、解答を伏せてもう一度。
- ▷ すべての問題がひと目で解き方がわかるようになるまで何度も取り組んでみましょう。

合成関数の微分

$$\left(f\left(g(x)\right)\right)' = f'\left(g(x)\right)\cdot g'(x)$$

- □ $\left(\sqrt{7x-1}\right)' = \frac{7}{2}(7x-1)^{-\frac{1}{2}}$
- □ $(\sin 3x)' = 3\cos 3x$
- □ $\left(\cos(2x-1)\right)' = -2\sin(2x-1)$
- □ $(\tan 2x)' = \frac{2}{\cos^2 2x}$
- □ $(\log|5x|)' = \frac{1}{x}$
- □ $\left(e^{x^2-3}\right)' = 2xe^{x^2-3}$

基礎編〜覚える積分公式〜

- □ $\int (x^2+x^{-1}+x^{-3})\,dx = \frac{1}{3}x^3 + \log|x| - \frac{1}{2}x^{-2} + C$
- □ $\int \sqrt[3]{x^2}\,dx = \int x^{\frac{2}{3}}\,dx = \frac{3}{5}x^{\frac{5}{3}} + C$
- □ $\int \sin x\,dx = -\cos x + C$
- □ $\int \cos x\,dx = \sin x + C$
- □ $\int \frac{1}{\cos^2 x}\,dx = \tan x + C$
- □ $\int \frac{1}{\sin^2 x}\,dx = -\frac{1}{\tan x} + C$
- □ $\int e^x\,dx = e^x + C$
- □ $\int 3^x\,dx = \frac{3^x}{\log 3} + C$

式変形から

- □ $\int \frac{x^2+x}{x^3}\,dx$
- □ $\int \frac{x^2+1}{\sqrt{x}}\,dx$
- □ $\int \sin^2 x\,dx$
- □ $\int \cos^2 x\,dx$
- □ $\int \sin^3 x\,dx$
- □ $\int \frac{x^2+x}{x-1}\,dx$
- □ $\int \frac{3x-2}{x^2-x}\,dx$

置換積分

- □ $\int e^{3x+1}\,dx$
- □ $\int (2x+3)^5\,dx$
- □ $\int \frac{1}{5x-1}\,dx$
- □ $\int x\sqrt{3x-2}\,dx$
- □ $\int \frac{2x}{\sqrt{2x+1}}\,dx$
- □ $\int (4x-1)(2x^2-x)^3\,dx$
- □ $\int \sin x\cos^7 x\,dx$
- □ $\int xe^{2x^2+1}\,dx$

log 型 *

$$\int \frac{g'(x)}{g(x)}\,dx = \log|g(x)| + C$$

- □ $\int \frac{x}{3x^2+2}\,dx$
- □ $\int \frac{x+3}{x^2+6x+5}\,dx$
- □ $\int \frac{\sin x - \cos x}{\sin x + \cos x}\,dx$
- □ $\int \tan x\,dx$
- □ $\int \frac{e^x}{e^x+4}\,dx$
- □ $\int \frac{1}{x\log x}\,dx$
- □ $\int \frac{1}{\tan x}\,dx$

部分積分

- □ $\int x\sin x\,dx$
- □ $\int xe^{-x}\,dx$
- □ $\int \log 5x\,dx$
- □ $\int \log|2x-3|\,dx$
- □ $\int \sqrt{x}\log x\,dx$
- □ $\int x^2\sin x\,dx$
- □ $\int e^x\cos x\,dx$

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* この形の積分のことを便宜的に log 型と書きました。置換積分の特別な形に過ぎませんが、この形を知っておくと計算が楽に早くできます:

$$\int \frac{g'(x)}{g(x)}\,dx = \int \frac{1}{t}\,dt = \log|g(x)| + C \qquad \begin{array}{c} t = g(x) \\ dt = g'(x)\,dx \end{array}$$

合成関数の微分 $\left(f\left(g(x)\right)\right)' = f'\left(g(x)\right)\cdot g'(x)$

□ $\left(\sqrt{7x-1}\right)' = \frac{1}{2}(7x-1)^{-\frac{1}{2}}\cdot(7x-1)' = \frac{7}{2}(7x-1)^{-\frac{1}{2}}$

□ $\left(\sin(3x)\right)' = \cos(3x)\cdot(3x)' = 3\cos 3x$

□ $\left(\cos(2x-1)\right)' = -\sin(2x-1)\cdot(2x-1)' = -2\sin(2x-1)$

□ $\left(\tan(2x)\right)' = \frac{1}{\cos^2(2x)}\cdot(2x)' = \frac{2}{\cos^2 2x}$

□ $\left(\log|5x|\right)' = \frac{1}{(5x)}\cdot(5x)' = \frac{1}{x}$

□ $\left(e^{(x^2-3)}\right)' = e^{(x^2-3)}\cdot(x^2-3)' = 2xe^{x^2-3}$

式変形から

□ $\int\frac{x^2+x}{x^3}\,dx = \int(x^{-1}+x^{-2})dx = \log|x| - x^{-1} + C$

□ $\int\frac{x^2+1}{\sqrt{x}}\,dx = \int(x^{\frac{3}{2}}+x^{-\frac{1}{2}})dx = \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + C$

□ $\int\sin^2 x\,dx = \int\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

□ $\int\cos^2 x\,dx = \int\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$

□ $\int\sin^3 x\,dx = \int(1-\cos^2 x)\sin x\,dx = -\cos x + \frac{1}{3}\cos^3 x + C$

□ $\int\frac{x^2+x}{x-1}\,dx = \int\left(x+2+\frac{2}{x-1}\right)dx = \frac{1}{2}x^2 + 2x + 2\log|x-1| + C$

□ $\int\frac{3x-2}{x^2-x}\,dx = \int\frac{3x-2}{x(x-1)}\,dx = \int\left(\frac{2}{x}+\frac{1}{x-1}\right)dx = 2\log|x| + \log|x-1| + C$ $\quad \frac{3x-2}{x(x-1)} = \frac{a}{x} + \frac{b}{x-1}$ (部分分数展開せよ)

置換積分

□ $\int e^{3x+1}dx = \int e^t\cdot\frac{1}{3}dt = \frac{1}{3}e^t + C = \frac{1}{3}e^{3x+1} + C$ $\quad t = 3x+1,\ dt = 3dx$

□ $\int(2x+3)^5 dx = \int t^5\cdot\frac{1}{2}dt = \frac{1}{2}\cdot\frac{1}{6}t^6 + C = \frac{1}{12}(2x+3)^6 + C$ $\quad t = 2x+3,\ dt = 2dx$

□ $\int\frac{1}{5x-1}\,dx = \int\frac{1}{t}\cdot\frac{1}{5}dt = \frac{1}{5}\log|t| + C = \frac{1}{5}\log|5x-1| + C$ $\quad t = 5x-1,\ dt = 5dx$

□ $\int x\sqrt{3x-2}\,dx = \int\frac{1}{3}(t+2)\sqrt{t}\cdot\frac{1}{3}dt = \frac{1}{9}\int(t^{\frac{3}{2}}+2t^{\frac{1}{2}})dt = \frac{1}{9}\left(\frac{2}{5}t^{\frac{5}{2}}+2\cdot\frac{2}{3}t^{\frac{3}{2}}\right) + C = \frac{2}{45}(3x-2)^{\frac{5}{2}} + \frac{4}{27}(3x-2)^{\frac{3}{2}} + C$ $\quad t = 3x-2,\ dt = 3dx,\ x = \frac{1}{3}(t+2)$

□ $\int\frac{2x}{\sqrt{2x+1}}\,dx = \int\frac{t-1}{\sqrt{t}}\cdot\frac{1}{2}dt = \frac{1}{2}\int(t^{\frac{1}{2}}-t^{-\frac{1}{2}})dt = \frac{1}{2}\left(\frac{2}{3}t^{\frac{3}{2}}-2t^{\frac{1}{2}}\right) + C = \frac{1}{3}(2x+1)^{\frac{3}{2}} - (2x+1)^{\frac{1}{2}} + C$ $\quad t = 2x+1,\ dt = 2dx,\ x = \frac{1}{2}(t-1)$

□ $\int(4x-1)(2x^2-x)^3 dx = \int t^3 dt = \frac{1}{4}t^4 + C = \frac{1}{4}(2x^2-x)^4 + C$ $\quad t = 2x^2-x,\ dt = (4x-1)dx$

□ $\int\sin x\cos^7 x\,dx = \int t^7\cdot(-dt) = -\frac{1}{8}t^8 + C = -\frac{1}{8}\cos^8 x + C$ $\quad t = \cos x,\ dt = -\sin x\,dx$

□ $\int xe^{2x^2+1}\,dx = \int e^t\cdot\frac{1}{4}dt = \frac{1}{4}e^t + C = \frac{1}{4}e^{2x^2+1} + C$ $\quad t = 2x^2+1,\ dt = 4xdx$

log 型 $\int\frac{g'(x)}{g(x)}\,dx = \log|g(x)| + C$

□ $\int\frac{x}{3x^2+2}\,dx = \int\frac{1}{6}\cdot\frac{6x}{3x^2+2}\,dx = \frac{1}{6}\int\frac{(3x^2+2)'}{3x^2+2}\,dx = \frac{1}{6}\log(3x^2+2) + C$

□ $\int\frac{x+3}{x^2+6x+5}\,dx = \int\frac{1}{2}\cdot\frac{2x+6}{x^2+6x+5}\,dx = \frac{1}{2}\int\frac{(x^2+6x+5)'}{x^2+6x+5}\,dx = \frac{1}{2}\log|x^2+6x+5| + C$

□ $\int\frac{\sin x-\cos x}{\sin x+\cos x}\,dx = \int\frac{-(\sin x+\cos x)'}{\sin x+\cos x}\,dx = -\log|\sin x+\cos x| + C$

□ $\int\tan x\,dx = \int\frac{\sin x}{\cos x}\,dx = \int\frac{-(\cos x)'}{\cos x}\,dx = -\log|\cos x| + C$

□ $\int\frac{e^x}{e^x+4}\,dx = \int\frac{(e^x+4)'}{e^x+4}\,dx = \log(e^x+4) + C$

□ $\int\frac{1}{x\log x}\,dx = \int\frac{\frac{1}{x}}{\log x}\,dx = \int\frac{(\log x)'}{\log x}\,dx = \log|\log x| + C$

□ $\int\frac{1}{\tan x}\,dx = \int\frac{\cos x}{\sin x}\,dx = \int\frac{(\sin x)'}{\sin x}\,dx = \log|\sin x| + C$

部分積分 $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

□ $\int x\sin x\,dx = \int x(-\cos x)'\,dx = x(-\cos x) - \int(x)'(-\cos x)\,dx = -x\cos x - \int 1\cdot(-\cos x)\,dx = -x\cos x + \sin x + C$

□ $\int xe^{-x}\,dx = \int x(-e^{-x})'dx = x(-e^{-x}) - \int(x)'(-e^{-x})dx = -xe^{-x} - \int 1\cdot(-e^{-x})dx = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + C$

□ $\int\log 5x\,dx = \int(x)'\log 5x\,dx = x\log 5x - \int x(\log 5x)'dx = x\log 5x - \int x\cdot\frac{1}{5x}\cdot 5\,dx = x\log 5x - \int dx = x\log 5x - x + C$

□ $\int\log|2x-3|\,dx = \int\frac{1}{2}(2x-3)'\log|2x-3|\,dx = \frac{1}{2}(2x-3)\log|2x-3| - \frac{1}{2}\int(2x-3)(\log|2x-3|)'dx$

$= \frac{1}{2}(2x-3)\log|2x-3| - \frac{1}{2}\int(2x-3)\cdot\frac{1}{2x-3}\cdot 2\,dx = \frac{1}{2}(2x-3)\log|2x-3| - \int dx = \frac{1}{2}(2x-3)\log|2x-3| - x + C$

□ $\int\sqrt{x}\log x\,dx = \int\left(\frac{2}{3}x^{\frac{3}{2}}\right)'\log x\,dx = \frac{2}{3}x^{\frac{3}{2}}\log x - \int\frac{2}{3}x^{\frac{3}{2}}\cdot\frac{1}{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\log x - \frac{2}{3}\int x^{\frac{1}{2}}\,dx = \frac{2}{3}x^{\frac{3}{2}}\log x - \frac{2}{3}\cdot\frac{2}{3}x^{\frac{3}{2}} + C = \frac{2}{3}x^{\frac{3}{2}}\log x - \frac{4}{9}x^{\frac{3}{2}} + C$

□ $\int x^2\sin x\,dx = \int x^2(-\cos x)'dx = x^2(-\cos x) - \int 2x(-\cos x)dx = -x^2\cos x + 2\int x\cos x dx = -x^2\cos x + 2\int x(\sin x)'dx$

$= -x^2\cos x + 2\left\{x\sin x - \int 1\cdot\sin x\,dx\right\} = -x^2\cos x + 2x\sin x + 2\cos x + C$

□ $I = \int e^x\cos x\,dx = \int e^x(\sin x)'dx = e^x\sin x - \int e^x\sin x\,dx = e^x\sin x - \int e^x(-\cos x)'dx = e^x\sin x - \left\{e^x(-\cos x) - \int e^x(-\cos x)dx\right\} = e^x\sin x + e^x\cos x - I$

$\therefore\ I = \frac{e^x\cos x + e^x\sin x}{2} + C$