基礎数学α 小課題第 21 回

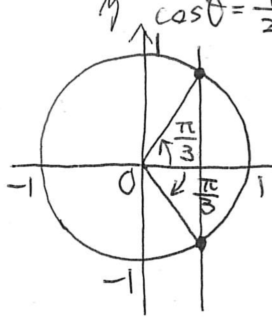
裏面にある略解をもとに丸付けをすること。授業の質問も書いてくれれば回答します。名前等、忘れずにていねいに書いてください！各問題の類題もあわせて示すようにしてみました。例・例題 と 節末 は教科書の該当する章の例・例題と節末問題を、14 などは問題集の番号を示しています。この課題の問題が解けなかったら教科書の例・例題に戻って確認、また、試験前には類題 (例の下にある練習問題も) も解いてみると良いでしょう。

1 年 ___ 科 ___ 番 氏名 ______________________

1. $0 \le \theta < 2\pi$ のとき、次の方程式・不等式を解け。pp.156-157, 例題 2 (p.163), 例題 3, 節末 7, 293, 294, 299, 300

(1) $2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

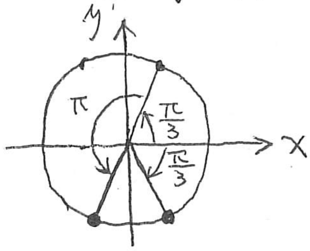


$\therefore\ \theta = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}$

$= \frac{\pi}{3},\ \frac{5}{3}\pi$

(2) $2\sin\theta = -\sqrt{3}$

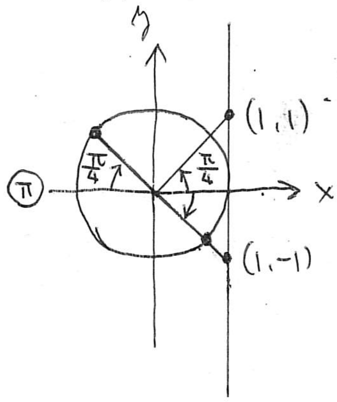
$\sin\theta = -\frac{\sqrt{3}}{2}$



$\therefore\ \theta = \frac{\pi}{3} + \pi,\ 2\pi - \frac{\pi}{3}$

$= \frac{4}{3}\pi,\ \frac{5}{3}\pi.$

(3) $\tan\theta = -1$



$\therefore\ \theta = \pi - \frac{\pi}{4},\ 2\pi - \frac{\pi}{4}$

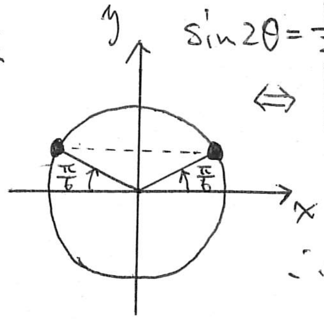
$= \frac{3}{4}\pi,\ \frac{7}{4}\pi.$

(4) $2\sin 2\theta = 1$ $(0 \le \theta < 2\pi)$

(Hint: $\alpha = 2\theta$ とおくと、$2\sin\alpha = 1$ $(0 \le \alpha < 4\pi)$)

2倍

$\sin 2\theta = \frac{1}{2}$ $(0 \le 2\theta < 4\pi)$ ←この範囲で！



$\Leftrightarrow\ \boxed{2\theta} = \frac{\pi}{6},\ \pi - \frac{\pi}{6},\ 2\pi + \frac{\pi}{6},\ 3\pi - \frac{\pi}{6}$

$2\theta = \frac{\pi}{6},\ \frac{5}{6}\pi,\ \frac{13}{6}\pi,\ \frac{17}{6}\pi$

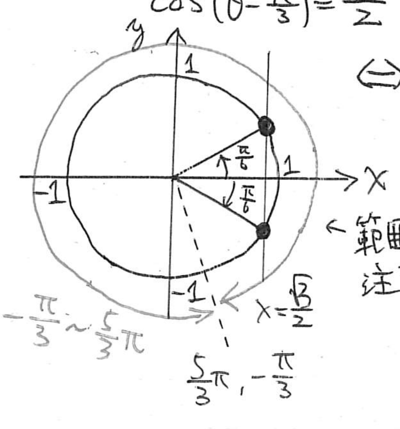
$\therefore\ \theta = \frac{\pi}{12},\ \frac{5}{12}\pi,\ \frac{13}{12}\pi,\ \frac{17}{12}\pi.$

(5) $2\cos\left(\theta - \frac{\pi}{3}\right) = \sqrt{3}$ $(0 \le \theta < 2\pi)$

(Hint: $\alpha = \theta - \frac{\pi}{3}$ とおくと、$2\cos\alpha = \sqrt{3}$ $(-\frac{\pi}{3} \le \alpha < \frac{5}{3}\pi)$)

$-\frac{\pi}{3}$

$\cos\left(\theta - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ $\left(-\frac{\pi}{3} \le \theta < \frac{5}{3}\pi\right)$



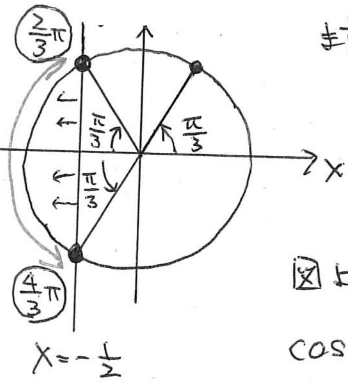
$\Leftrightarrow\ \boxed{\theta - \frac{\pi}{3}} = -\frac{\pi}{6},\ \frac{\pi}{6}$

ひとつと見る

$\theta = \frac{\pi}{3} - \frac{\pi}{6},\ \frac{\pi}{3} + \frac{\pi}{6}$

$= \frac{\pi}{6},\ \frac{\pi}{2}.$

(6) $2\cos\theta < -1$ $\Leftrightarrow$ $\cos\theta < -\frac{1}{2}$



まず、$\cos\theta = -\frac{1}{2}$ を解く.

$\cos\theta = -\frac{1}{2}$

$\Leftrightarrow\ \theta = \pi - \frac{\pi}{3},\ \pi + \frac{\pi}{3}$

$= \frac{2}{3}\pi,\ \frac{4}{3}\pi$

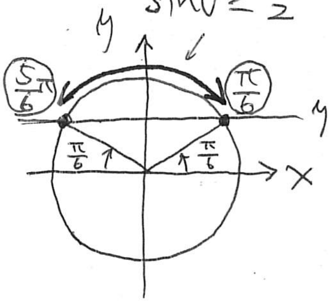
図より,

$\cos\theta < -\frac{1}{2}$

$\Leftrightarrow\ \frac{2}{3}\pi < \theta < \frac{4}{3}\pi.$

(7) $2\sin\theta \geq 1$

$\sin\theta \geq \frac{1}{2}$



$y \geq \frac{1}{2}$は上側

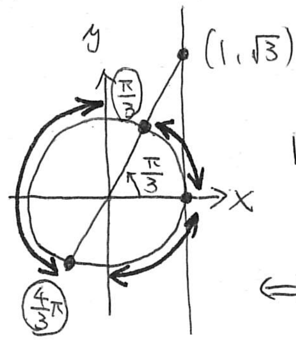
$\sin\theta = \frac{1}{2}$

$\iff \theta = \frac{\pi}{6}, \frac{5}{6}\pi$

$\therefore \sin\theta \geq \frac{1}{2}$

$\iff \frac{\pi}{6} \leq \theta \leq \frac{5}{6}\pi$

(8) $\tan\theta < \sqrt{3}$



$\tan\theta = \sqrt{3}$

$\iff \theta = \frac{\pi}{3}, \frac{4}{3}\pi$

図より，

$\tan\theta < \sqrt{3}$

$\iff 0 \leq \theta < \frac{\pi}{3}, \frac{\pi}{2} < \theta < \frac{4}{3}\pi,$

$\frac{3}{2} < \theta < 2\pi$

2. 加法定理を用いて、$\sin 15°$, $\cos 15°$, $\tan 15°$ を求めよ。例 1 (p.170), 306

$$\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos 15° = \cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$
$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

有理化

$$\tan 15° = \frac{\sin 15°}{\cos 15°} = \frac{\frac{\sqrt{6} - \sqrt{2}}{4}}{\frac{\sqrt{6} + \sqrt{2}}{4}} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = \frac{(\sqrt{6} - \sqrt{2})(\sqrt{6} - \sqrt{2})}{(\sqrt{6} + \sqrt{2})(\sqrt{6} - \sqrt{2})}$$
$$= \frac{6 - 2\sqrt{12} + 2}{6 - 2}$$
$$= 2 - \sqrt{3}$$

加法定理

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

略解

1. (1) $\frac{\pi}{3}, \frac{5}{3}\pi$ (2) $\frac{4}{3}\pi, \frac{5}{3}\pi$ (3) $\frac{3}{4}\pi, \frac{7}{4}\pi$ (4) $\frac{\pi}{12}, \frac{5}{12}\pi, \frac{13}{12}\pi, \frac{17}{12}\pi$ (5) $\frac{\pi}{6}, \frac{\pi}{2}$ (6) $\frac{2}{3}\pi < \theta < \frac{4}{3}\pi$
(7) $0 \leq \theta \leq \frac{\pi}{6}, \frac{11}{6}\pi \leq \theta < 2\pi$ (8) $0 \leq \theta < \frac{\pi}{3}, \frac{\pi}{2} < \theta < \frac{4}{3}\pi, \frac{3}{2}\pi < \theta < 2\pi$

2. $\sin 15° = \frac{\sqrt{6} - \sqrt{2}}{4}$, $\cos 15° = \frac{\sqrt{6} + \sqrt{2}}{4}$, $\tan 15° = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = 2 - \sqrt{3}$